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INTERNATIONAL BLACK SEA UNIVERSITY
FACULTY of BUSINESS and TECHNOLOGIES
BUSINESS ADMINISTRATION DOCTORAL PROGRAM

Developing a Method for Optimization of Assortment Structure for a Competitive Firm

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Extended Abstract of Doctoral Dissertation in Business Administration

Tbilisi, 2020

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INTRODUCTION

In a real market, managers have to carry out complex analysis of macro and microeconomic environment. They have to evaluate simultaneously hundreds of economic parameters to develop short run operating and long run development strategies.

The ultimate goal of each business is to maximize profit. Manipulating with the prices is one of available strategies of Profit Maximization of firms in non-competitive markets. For competitive firms this strategy is mostly useless and unpractical. Another strategy is related to the right selection of products or products assortment optimization strategy. Both of these strategies require complex analysis of the economic parameters.

Problem Statement

When the business strategy is price-optimization, managers need to make pricing decision by considering several products at the same time. By doing so, decision makers can control for substitution effects and benefit from potential synergies between the products (Goic, 2011). The price optimization models for one product are well known, but they cannot be used in a multi-product case, because of an effect of a cross-elasticity interactions among the products (Gallego & Wang, 2014). The latter leads to complicate mutual correlations among products' prices and demands, and therefore, requires usage of alternative approaches.

When the business strategy is revenue-optimization or, more precisely, revenue-maximization, assortment optimization becomes one of the main objectives of Revenue Management. Indeed, development of methods for retail assortment analysis and its optimization is highly important. On one hand, if it is done correctly and on time, it can result in a significant increase in a company's revenues and improvement of overall performance. On the other hand, failure to succeed in the analysis, results in a waste of capacities and resources and as a consequence, may end up with a huge loss or bankruptcy of the business.

When capacities are limited, decisions regarding optimal usage of resources play a vital role, as they are directly associated with the costs. In case of physical retailing, it is strategically important to decide what to offer customers, e.g. due to the limited shelves space. In case of e-commerce, the problem of assortment optimization gives rise to a new challenge of maximizing total revenue, through dynamic observation on the real-time data and by tailoring more personalized approaches, e-commerce maximizes its revenues.

There is lack of publications in regards to assortment optimization problem. Existed publications focus on step by step integration of real market constraints into the model. However, introducing a large number of variables into the model results in computational complexity and Nondeterministic Polynomial Time (NP) Completeness. In other words, when the number of variables in the model increases, the time required for solving a given problem with any known algorithm is also gradually increasing. As a result, determining whether the given problem is tractable or not is impossible and the problem becomes unsolvable.

Thus, existed literature provides a good theoretical framework for solving several issues of Revenue Management. However, when going to the application, proposed models become useless. Limitations of the existing models and the lack of their practical applicability for a real market, stimulates the further studies in this field.

Goal Statement

The general goal of the study is to develop decision-making support tool for managers of retail firms in the area of Multi-Product Assortment Structure Optimization as a part of Revenue Management.

Based on the generated data, the study formulates twofold goals:

- to develop simple, reliable and efficient methods of retail assortment analysis and its optimization,
- to suggest a model which helps to evaluate given economic parameters for the optimal decision-making.

Therefore, the dissertation addresses the problem of retail assortment structure optimization by incorporating several real market constraints into the model. Specifically, the effect of substitution and complementarity among products on the structure of assortment is considered.

Research Questions

The following research questions are formulated in the dissertation:

RQ1. What kind of retail assortment optimization models are developed in the existing literature? What are their limitations?

RQ2. What should be the structure of statistical models for retail assortment planning and optimization taking into account the relationships between the goods (complementary/substitutionary)?

RQ3. How to estimate the relationship effects among substitute and complementary products on the retail assortment structure?

RQ4. What are the functional relationships between the retail assortment optimal structure and substitute-complementarity factors (products)?

RQ5. What is the market status, productivity and associated risk of a particular product within a given assortment?

Research Objectives

To answer the research questions and to achieve the research goals, the study objectives are formulated as follows:

1. To determine structure of statistical methods and models for retail assortment planning and optimization;
2. To elaborate the methods of estimating relationship effects among substitute and complementary products on the retail assortment structure;
3. To determine functional relationships between the retail assortment optimal structure and substitute-complementarity factors (products);
4. To determine the market status, productivity and associated risk of a particular product within a given assortment;
5. To develop an assortment marketable portrait based on the predefined parameters;
6. To develop easy, user-friendly Retail Assortment Optimization Software for managers.

Novelty and Actuality

Based on existing Single-Product Retail Assortment Optimization (SP_RPO) model and by generalizing this model, the dissertation presents a new decision-making support tool in total revenue maximization problem. The study suggests an alternative model of Multi-Product Retail Assortment Optimization (MP_RAO), which is a novel approach in academic literature.

The model allows to undertake comprehensive analysis of economic, financial and managerial aspects of the retail business. The model may be used as a supportive managerial decision-making tool for evaluating the potential profitability of a single product as well as assortment of products in a specific market.

Using accumulated statistical data of the prices and demands of the products, a corresponding software was developed.

Practical Value

With the model and Retail Assortment Optimization Software for managers, suggested in the dissertation, managers can easily apply existed data and generate the valuable results for further interpretation and decision making. Assortment optimization will benefit both sides, consumers who find the needed goods and retail firms that can achieve short run profitability and long run sustainability.

Theoretical Value

Deep analysis of the literature as well as the dissertation findings (new concepts, alternative approaches and a new model) contribute to further development of Revenue Management and Decision Support Systems (DSS) that is an integrated part of Managerial Decision Making.

The dissertation creates incentives for further studies in this area.

Research Methods

Research methods incorporated in the dissertation are the following:

- ✓ Critical analysis of the existed literature;
- ✓ Application of methods of multivariate statistical analysis, in particular - methods of statistical simulation of retail assortment demand and price observations;
- ✓ Multivariate linear regression analysis, generalized partial regression, methods of graphical analysis;
- ✓ Numerical verification of models based on statistical modeling technique.

Structure of the Dissertation

Dissertation consists of Introduction, Literature Review, Theoretical part of elaborating the models, and the Practical Demonstration of the model, Conclusion, 42 Figures, 48 Tables and 120 References. The author of the dissertation has published 4 scientific articles, which are focused on various aspects to the research topic. All of them are related to the dissertation topic.

CHAPTER 1. LITERATURE REVIEW

The dissertation is focused on the topic of retail product assortment structure optimization in competitive firms as a tool of revenue management. Literature review provides a summary of theoretical research that has been conducted in this area and critically analyzes the existed methods. At the initial stage of literature review different pricing methods as the tools of revenue management are provided. First of all, general models of pricing are introduced and the study goes further in analyzing static and dynamic, deterministic, stochastic and supply based pricing models. This part is concluded with highlighting the nonlinear pricing models and their practical applications.

The second part of the literature review provides the critical analyzes of existed product assortment optimization methods in revenue management problem by dividing discussion into four main sections: 1) General Approaches to the Assortment Optimization Problem; 2) Choice-Based Models for Network Revenue Management Problem; 3) Discreate Choice Models for Assortment Optimization Problem; 4) Assortment Optimization Under Substitution;

Based on the analyses the dissertation aims to develop decision making support tool for managers to effectively manipulate with assortment structure optimization and to make the business more profitable.

CHAPTER 2. THEORETICAL FOUNDATIONS OF A MULTI-PRODUCT RETAIL ASSORTMENT STRUCTURE OPTIMIZATION MODELS

Second chapter of the dissertation is focused on the Theoretical Foundations of a Multi-Product Retail Assortment Structure Optimization Models. At the beginning of the chapter Single Product Retail Assortment Optimization (SP_RAO) Problem is discussed. Despite of the usefulness of the SP_RAO model, it cannot be applied when a manager faces Multi-Product Retail Assortment Optimization (MP_RAO) problem. Besides that, the latter problem differs from SP_RAO problem with high dimensions of the all functions represented in SP_RAO model, it has another specific issue: both prices and demands, being random variables, may exhibit different covariate relationships, determined by means of substitution and complementation processes. Such kind of covariate relationships creates a system of constraints which should is included in the relevant mathematical model of the MP_RAO problem developed in the dissertation. Further sections of the second chapter are dedicated to determination of the model, identification of the parameters of this model and then to the theory of MP_RAO Model is formulated.

Multivariable Optimization

Considering the demands as factors impacting the process we introduce the following terms.

We call demands with index coinciding the certain index of the price of the i-th item Direct Factors.

We call other demands dj ($\boldsymbol{i \neq j}$; $\boldsymbol{j = 1, 2, \dots, m}$)) Indirect Factors.

We call the sum of indirect factors $\boldsymbol{\sigma_i = \sum_{j=1, j\neq i}^{m} \gamma_{ij} d_j}$ Resultant of Indirect Factors or just Resultant Factor.

As it was mentioned above, the resultant factor $\boldsymbol{\sigma_i}$ is a function of m-1 variables γ_{ij} (j=1, 2, …, m; $j \neq i$).

Remark 1. One could use more suggestive terms like endogenous (for direct factors) and exogenous (for indirect factors), but because the terms are conventionally used in Econometrics, we decided (to avoid confusion) to use the terms introduced above. At the same time, we would like to note that direct factors have endogenous nature and indirect ones – exogenous.

Let's consider now separately one equation i $p_i = \gamma_{i1} d_1 + \cdots + \gamma_{im} d_m + b_i + \varepsilon_i$ (1) from the system (2)

$$\begin{aligned} p_1 &= \gamma_{11} d_1 + \ldots + \gamma_{1m} d_m + b_1 + \varepsilon_1 \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots , \\ p_m &= \gamma_{m1} d_1 + \ldots + \gamma_{mm} d_m + b_m + \varepsilon_m \end{aligned} \qquad (2)$$

Where p_i – price of the item i;

d_i - demand of the item i;

ε_i - normally distributed random variable with zero mean corresponding to observations of item i.

Observe, that γ_{ij} (i, j=1,2,…,m) now may have positive or negative signs, as demands can be in different relations (substitute or complement) with different prices.

Using the resultant term, the equation can be rewritten as follows a **generalized partial regression,** (Abdi, Dowling, Valentin, Edelman, & M., 2002)**,** (Draper & Smith, 1998)

$$p_i = \gamma_{ii} d_i + \sigma_i + b_i + \varepsilon_i \quad . \tag{3}$$

Further, assuming estimation (by means of classical LS (see previous section)) all of the parameters γ_{ij}(i, j = 1,2, … , m), one can rewrite (3) without random term

$$\widehat{p_i} = \gamma_{ii} d_i + \sigma_i + \hat{b}_i \ , \tag{4}$$

where $\widehat{p_i}$ and $\hat{b}_i$ -are estimated (predicted) price and corresponding intercept.

Now (4) can be used for optimization in the terms of the section 2.1. Again, important to note that $\gamma_{ii} < 0$, $\hat{b}_i$>0 and resultant factor δ_i, being a function of indirect factors (d_{ij} $(i \neq j;\ i, j = 1,2, \dots, m)$), can be negative, zero or positive (any real number). Clear, that latter is depend on concrete combination of d_{ij} and estimated γ_{ij} $(i \neq j;\ i, j = 1,2, \dots, m)$.

Total Revenue of an item i can be represented now as

$$TR_i = \hat{b}_i d_i + \sigma_i d_i + \gamma_{ii} d_i^2 = (\hat{b}_i + \sigma_i) d_i + \gamma_{ii} d_i^2 . \tag{5}$$

One can call the (5) the second order parametric multinomial, because the term δ_i is the function of m-1 indirect factors.

It is easy to obtain values of prices and demand which provide maximum of Total Revenue of item i. To find maximum of the function TR=bp+γp^2 one must differentiate it:

$$\frac{dTR_i}{dq_i} = \hat{b}_i + \sigma_i + 2\gamma_{ii} d_i$$

Making it equal to zero gives simple equation

$$\hat{b}_i + \sigma_i + 2\gamma_{ii} d_i = 0,$$

which leads to optimal value of the demand variable, that is to the demand value of i-th item which maximizes i-th Total revenue function.

$$d_i^{op} = -\frac{\hat{b}_i + \sigma_i}{2\gamma_{ii}}; \tag{6}$$

Substituting the later into (5) implies corresponding value of optimal Price of i-th item

$$\hat{p}_i^{op} = \frac{\hat{b}_i + \sigma_i}{2}. \tag{7}$$

The production of (6) and (7) defines the maximum value of the Total Revenue function of i-th item

$$Tr_i^{max} = -\frac{(\hat{b}_i + \sigma_i)^2}{4\gamma_{ii}} . \tag{8}$$

The obtained expressions require comments. All the values depend on the resultant of the indirect factors δ_i, so they are actually function of m-1 indirect variables. One of them is critically dependent on the resultant: as the value of the resultant can be equal to any real number (depending of the certain combination of the all indirect factors), the optimal price p_i^{op} in (7) can become negative. It means that including indirect effects in the optimization model changes evaluation of business effectiveness of the item (or product): optimal values, calculated without considering indirect factors, may be actually unacceptable.

The next step is including costs in the model.

The total cost function of the i-th item can be represented by means of straight-line equation

$$C_i^T = C_i^F + c_i^v d_i, \quad (9)$$

where C_i^F-fixed cost of the i-th item;

and c_i^v – variable cost of the i-th item.

Now we solve optimization problem with respect to Profit function

$$Prt_i = TR_i - C_i^T = (\hat{b}_i + \sigma_i)d_i + \gamma_{ii}d_i^2 - C_i^F - c_i^v d_i. \quad (10)$$

Especial interest has the question about a range of profitability of the i-th item, which is defined by means of break-even points. They are defined as roots of the following quadratic equation

$$(\hat{b}_i + \sigma_i)d_i + \gamma_{ii}d_i^2 - C_i^F + c_i^v d_i = \gamma_{ii}d_i^2 + (\hat{b}_i + \sigma_i - c_i^v)d_i - C_i^F = 0$$

The latter is a simple quadratic equation (with respect to unknown d_i), which can be easily solved

$$\pi_{1,2}^i = \frac{-(\hat{b}_i + \sigma_i - c_i^v) \pm \sqrt{(\hat{b}_i + \sigma_i - c_i^v)^2 + 4C_i^F\gamma_{ii}}}{2\gamma_{ii}}. \quad (11)$$

Let's call a value

$$\Delta = \left|\boldsymbol{\pi}_1^i - \boldsymbol{\pi}_2^i\right| \quad (12)$$

range of profitability.

Reason for it is clear: for any d_i such that $\min(\pi_1^i, \pi_2^i) \le d_i \le \max(\pi_1^i, \pi_2^i)$, $Prt_i \ge 0$.

It should be noted again, that Δ is a function of m-1 indirect variables, via indirect' resultant δ_i, and the interval may not exist for some values of resultant δ_i. Even in SP_RAO model existence of Δ depends on values of b_i, c_i^v, C_i^F *and* γ_{ii}, but now (11) reveals new factor – Indirect

Resultant σ_i. Below we shall speak in detail about various system constraints emergent due to the impact of indirect factors.

Also, it is easy to calculate the rest of optimal parameters. The method is same as it was in SP_RAO model. Differentiating of (10) gives

$$p_i^{op} = -\frac{\hat{b}_i+\sigma_i+c_i^v}{2\gamma_{ii}}, \qquad (13)$$

$$d_i^{op} = -\frac{\hat{b}_i+\sigma_i+c_i^v}{2}, \qquad (14)$$

and

$$Prt_i^{max} = -\frac{(\widehat{b}_i+\sigma_i)^2-(c_i^v)^2}{4\gamma_{ii}} \qquad (15)$$

One can see that all the above expressions are also functions of indirect variables.

We call (6) ÷ (8) and (11) ÷ (15) optimal structure of i-th item.

Impact of Indirect Factors Resultant on the Value of i-th Item's Market Sustainability

Conceptions of Item Sustainability, Market Profitability and Market Risk

In this section we are introducing important conception of i-th item's **market sustainability.**
Let n_i^p be a number of the values of the observed demand of the i-th item fall within the range of profitability (12)

$$\pi_1^i < d_j^i < \pi_2^i \text{ (j=1, 2, …, } n_i^p\text{)}. \qquad (16)$$

We used strict inequalities, because we consider that the cases when the observed demand value equals to the one of the end points of the range of profitability $\left(d_j^i = \pi_1^i \ or \ d_j^i = \pi_2^i\right)$ is not interesting for analysis.

If n_i is total number of observations of the demand of the i-item (direct factor of i-th item), then $n_i - n_i^p$– number of observations fall out of the range of profitability.

Definition 1. Index of Market Profitability (IMP) of the i-th item is the ratio

$$\boldsymbol{\eta_i = \frac{n_i^p}{n_i}.} \qquad \textbf{(17)}$$

The definition implies that

$$0 \le \eta_i \le 1. \qquad (18)$$

One can consider index of profitability as an empirical probability that i-th item will remain within the range of profitability. When η_i=1 item i is absolutely profitable or riskless.

Definition 2. Index of market risk (IMR) of the i-th item is the value

$$\boldsymbol{\rho_i = 1 - \eta_i}. \qquad \textbf{(19)}$$

From the Definitions 1 and 2 it follows that IMR of the item i is low or zero only and only if majority or all of observations are within the range of the profitability. One must underline the following: 1) We suggested the new measure of risk, which is not conventionally equal to standard deviations, but is, by its nature, probability (empirical), which is more universal measure than standard deviation.

Let's ρ_i^c denote critical value of the IMR of the i-th item, then we can introduce the following:

Definition 3. We call the i-th item Market Sustainable (MS) if $\boldsymbol{\rho_i \le \rho_i^c}$.

Unfortunately, threshold value ρ_i^ccan not be detected formally, it should be done just intuitively. Of course, the value can be chosen with respect to certain peculiarities of the item.

CHAPTER 3. QUANTITATIVE EXAMPLES

Third chapter of the dissertation is dedicated to the quantitative demonstration of the model. For this purpose, a specially generated set of data was used. With this in view a special method of relevant data generation was developed.

Data Generation Methodology

Theoretical Foundations of Data Generation Methodology

Notations and Assumptions.

D-a n×m matrix of observed predictors, where n -number of observations, m- number of predictors;

p- n-vector-column of observed values of dependent (predicted) variable;

$\bar{p}$–predefined mean of random vector p;

σ- predefined standard deviation of random vector p;

μ_i –predefined mean of i-column of D;

σi- predefined standard deviation of i-column of D;

Rdd - m×m correlation matrix of paired correlations coefficients between pairs of predictors;

Rpd – m-vector-column of paired correlations coefficients between variable P and pairs of predictors.

The method is based on assumption that the linear relations (1) holds

$$p = \gamma_1 d_1 + \cdots + \gamma_m d_m + \bar{p} + \varepsilon, \qquad (20)$$

where ε- normally distributed random variables with zero mean and identically constant dispersions σ.

Now objective of the method can be formulated as follow:

Given correlation matrix Rdd , vector Rpd , ε_i normally distributed random variables, define D matrix of predictors and p vector of the dependent variable[1], bounded with D by means of (20).

One must underline that m random variables are bounded by means of internal correlative structure defined by the matrix Rdd, whereas p vector is bounded with D by means of Rpd m-vector-column of paired correlations coefficients. The latter means that one has to generate regression coefficients γ_i (i-1,2,…,m) which are completely defined by the matrixes Rdd and Rpd.

Considering that Rdd is correlation matrix, that is positively definite symmetric matrix, it can be factorized by means of Cholesky decomposition (Lang, 2002), (Trefethen & David Bau, 1997)

Rdd=LLT, (21)

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[1] Regression equation (20) is almost identical to (1): unlike (1) it represents only one dependence for the certain good (no index i).

where L is a m×m lower triangular matrix with real and positive diagonal entries, and LT denotes the transpose of L.

Proposition. Let H - be an n×m matrix of uncorrelated standard (with zero mean and standard deviation equal to 1) variables, that is HHT=I, where I-is identity matrix, then the matrix (it is actually is linear transformation of H variables)

X=LH (22)

is the n×m matrix with internal correlative structure defined by the matrix Rdd.

Proof is easy. Substitution of (22) in (21) gives

LHHTLT=LLT=D.

Q.E.D.

It was mentioned above, that to create the matrix X, the matrix H with feature HHT=I should be used. The latter can be done like the following: create m random variable, of n numbers each (the n×m matrix X1), then apply to the matrix: 1. the Gram–Schmidt orthogonalization process and 2. Standardization of the result of the orthogonalization. Finally obtained matrix H will satisfies necessary requirements HHT=I. Note, that randomness of the m n-dimensional variables provides their linear independence, that justifies correctness of usage of the Gram–Schmidt orthogonalization process.

Note that matrix X is not standardized and cannot be identified with the desired matrix D. The only common thing with D is that its columns are bounded by partial correlation coefficient defined in Rdd. To generate the matrix D one has to make the following simple transformations:

1. Standardizing each column of the matrix X, that is transforming X to Xst, where each column is standard variable with zero mean and standard deviation equals to 1;
2. And then back to natural values defined by predefined values of $\sigma_i \, and \, \mu_i \, (i = 1,2, \ldots, m)$: $d_{ij} = \sigma_j xst_{ij} + \mu_j$(i=1,2,…,n; j=1,2,…,m) for each column.

The next step is generating the regression coefficients γ_i (i=1, 2 ,…, m). First one has to detect their standardized counterparts γs_i (i=1, 2 ,…, m) , which can be found as solutions of the following linear system

$R_{dd}\gamma s_i = R_{pd}$,

from the later

$\gamma s_i = R_{dd}^{-1}$ R_{pd}. (23)

Thus, natural values of the desired regression coefficients will be

$\gamma_i = \gamma s_i \frac{\sigma}{\sigma_i}$. (24)

The last step is generating vector P by means of regression equation (20)

$p = D\gamma + \bar{p} + \varepsilon$. (25)

Thus, the represented method completely solves the problem defined in the definition of its objectives, represented above.

Data was generated for five products: five demands and five prices.

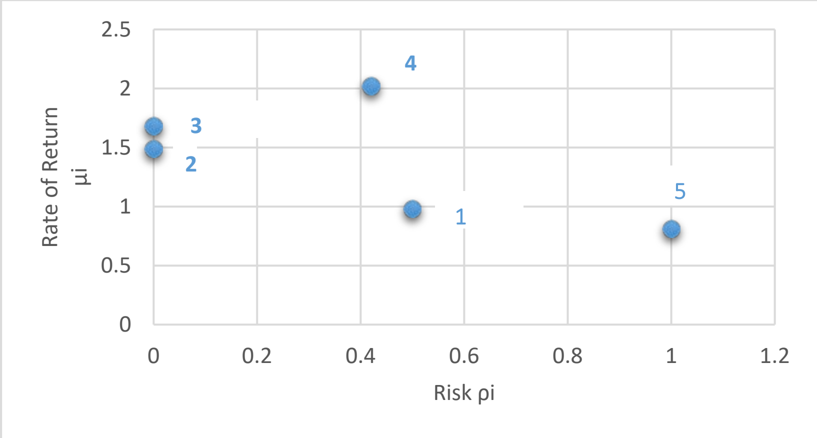
After demonstrating the first product's numerical results, the following sub-section of the third chapter illustrates the summarized Market Potential and Marketable Portrait of the Entire Assortment of Products. The table below shows these results for five products:

Table 1. Data for Marketable Portrait of the Assortment

	Item1	Item2	Item3	Item4	Item5
Variable Cost, C_v	11.19	19.35	27.18	18.12	18.39
Fixed Cost, C_f	275.69	937.43	1449.20	2507.60	547.58
Expected Costs	723.31	1711.60	2586.50	3559.30	1706.50
Expected Resultant	-6.38	18.78	23.89	7.57	2.27
Risk	0.50	0.00	0.00	0.42	1.00
Rate of Return	0.98	1.49	1.68	2.02	0.81

The graphical illustration of the Marketable Portrait of the Assortment shown above in the table is shown in the figure:

Fig. 1. *Marketable Portrait of the assortment described by a system of regression equations*



The Portrait on Fig. 1. clearly shows that items 2 and 3 are marketably sustainable (low risks and high rate of returns), items 1 and 5 are marketably unsustainable because of the high risk and low rate of return, and item 4 requires additional study (close to partially efficient products' category): the highest level of the rate of return, but nonzero risk (0.42) close to chosen critical value – 0.5. All of the considerations should be used for final evaluation of the assortment.

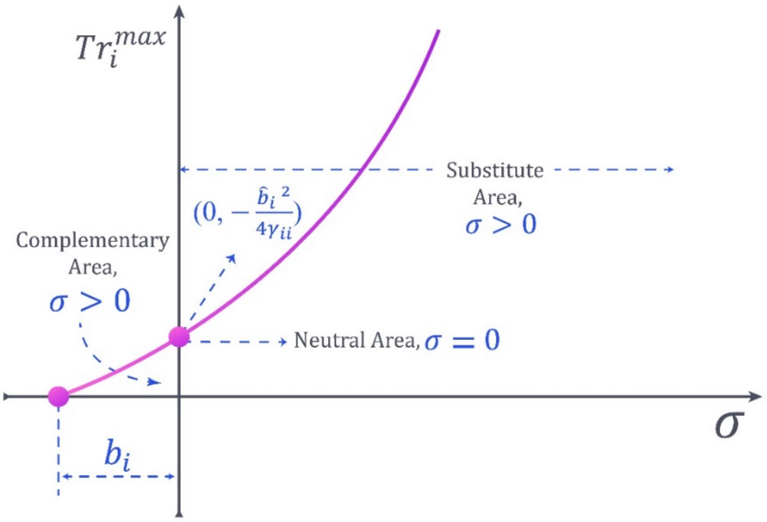
As mentioned in the previous chapters, manager has an overall picture of the assortment's market profitability potential and based on this data she/he can make the relevant judgements and decisions.

The rest of the third provides practical demonstration of the model for the rest of the products. It shows numerical results with graphical illustrations and provides the ways of applications for managers.

Summary of important outcomes

The purpose of the study was to develop a decision-making support tool for managers in particular in the field of product assortment structure optimization. For this purpose, the mathematical model was developed and number of different parameters (including p_i^{op}, d_i^{op}, Prt_i^{max}) were introduced. Based on these parameters one can evaluate the optimal structure of a particular product into a given assortment of products, as well as the circumstances of the entire assortment. The fundamental conception introduced in the dissertation is the *Resultant of Indirect Factors* (σ), which is the measure of overall influence of products in a given assortment over a particular product within the same assortment. As shown later in the study, it has a very significant impact on the optimal structure of the products and correspondingly on the revenue. Figures shown below represent the graphical demonstration of this interconnection:

Fig. 2 Tr_i^{max} *As the Function of Indirect Resultant*



Further findings of the dissertation introduce the new conceptions of *Item Sustainability*, *Market Profitability and Market Risk.*

As a final result, which is very handy for managers to evaluate the products *profitability potential* conceptions of *Marketable State of the Item* and *Marketable Portrait of a Stock /Assortment* were introduced. The later is based on the two main parameters introduced in the model – products risk $(\boldsymbol{\rho_i})$ and the rate of return $(\boldsymbol{\mu_i})$. The figure below shows the marketable state of a product based on these two parameters:

Fig. 3. *Marketable Space and Item Representation States*

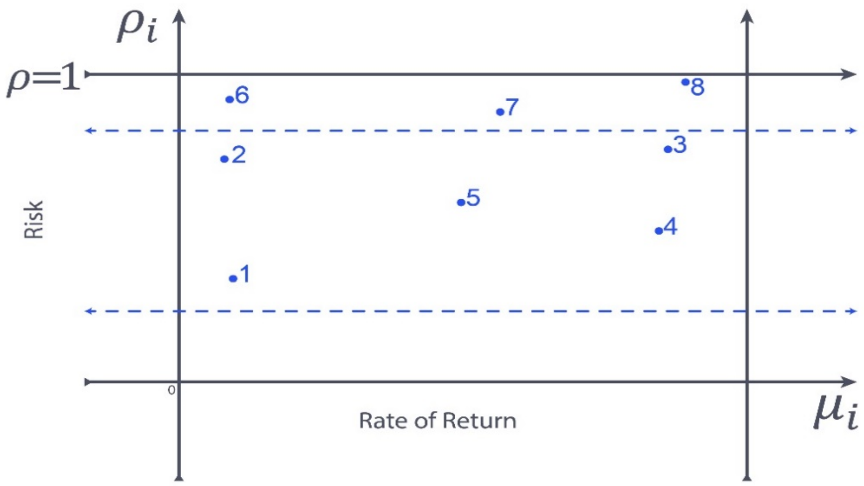


Fig. 3 - 1- item with low risk and low rate of return; 2- item with high risk and low rate of return; 3- item with high risk and high rate of return; 4- item with low risk and high rate of return; 5- item with average risk and average rate of return; 6,7,8 -items with different rate of returns, but not Sustainable: $\rho_i \geq \rho_i^c$.

The picture gives easy graphical method of evaluation of the given item in the assortment: items with high risk and the low expected rate of return can be excluded from the stock. The latter can be considered as a new method of the managerial decision making.

The idea of the state of the item leads to the conception of *Marketable Portrait of a Stock/ Assortment*.

Fig. 4. *Marketable Portrait of a Stock/ Assortment*.

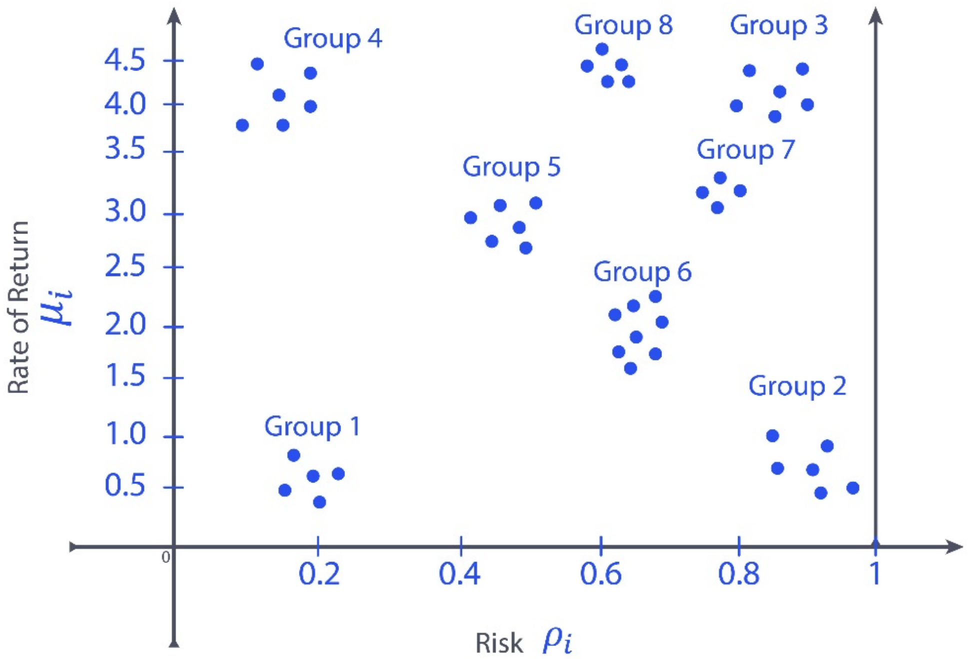


Fig. 4 - Marketable Portrait of a Stock. Group of the point 1- items with low risk and low rate of return; Group of the points 2- items with high risk and low rate of return; Group of the points 3- items with high risk and high rate of return; Group of the points 4- items with low risk and high rate of return; Group of the points 5- items with average risk and average rate of return; 6,7,8 - items with different rate of returns, but not Sustainable: $\rho_i \geq \rho_i^c$.

This figure gives generalization of the previous figure and we can repeat what we said about the state of the one item: the picture gives easy graphical method of evaluation of the given set of the items of entire stock, and therefore can also be used as new method of the managerial decision making, which allows improving of the marketable structure of the stock.

CONCLUSIONS

To summarize the results and findings of the dissertation, we have critically analyzed the scientific literature in the field of Revenue Maximization Problem, in particular, we have discussed assortment optimization techniques with a single product consideration, as well as the multiple product approaches.

RQ1. Critical analysis of the literature highlighted the drawbacks of existing models, especially in a multi-product case, where the number of real market constraints are not included, and in some cases, when they are incorporated into the models, those models face the problem of computational complexity. Therefore, the existed models face the problem of practical implementation.

In our work, we have developed a new approach to assortment optimization problem and we made the following conclusions:

1. It was shown that a set of regression equations of the MP_RAO problem can be considered as the seemingly unrelated set of equations.
2. It was proved that the set of seemingly unrelated equations can be estimated separately by means of Classical Least Square method.
3. The conception of Generalized Partial Regression was introduced.
4. The conception of the Resultant of Indirect factors was introduced.
5. The following conceptions were introduced:
    a. the hyperplane of the latent impacts of indirect factors and conditions for certain values of indirect factors for which the impact of indirect resultant vanishes, and Total Revenues maximums of paired and generalized partial regressions can be equal;
    b. economically feasible area of item i;
    c. index of market profitability of the i-th item;
    d. index of market risk of the i-th item;
    e. the i-th item's Market Sustainability;
    f. the i-th item's range of profitability;
    g. Marketable State S_i of the i-th item;
    h. Marketable Space;
    i. Marketable Portrait of a Stock.

RQ2. Above conceptions allowed determining of the Optimal Market Characteristics of the item as functions of direct and indirect factors.
RQ3. All of the above allow:

a) efficient using of the forecasting of prices of goods of the stock;

b) Creating a simple graphical method of evaluation of optimal structure of a retail assortment.

Besides, a special method to generate a finite set of multi-normally distributed random variables with predefined correlations matrix was developed. On the base of developed method, a set of prices and demands of five substitute/complementary products were generated
RQ4 and **RQ5.** We have calculated following economic parameters of those five products to show the overall marketable portrait of a product or of the assortment: Dop, Pop, TR max, Range of

Profitability, Profit Maximum, Index of market profitability (IMP), Index of market risk (IMR), Expected rate of return of the product.

RECOMMENDATIONS

A special easy, user-friendly Retail Assortment Optimization Software was developed as a decision-making support tool for managers. The developed model and approaches can be used for large, medium and small retail firms.

Possible Directions for Further Study

From practical point of view linear dependence between demand and price is not always the case, so the non-linear dependence can be considered as more reliable and practical. At the same time, estimation of parameters of non-linear models requires usage of special methods, like nonparametric or/and kernel approaches. This requires relevant changes in the products' marketable characteristics and development of special software.

Publications related to the dissertation:

Lekishvili, T. & Datuashvili, D., (2020). An Algorithm of Simulatin of Set of random Variables with Predefined Correlation Structure. *Journal of Technical Science and Technologies, 7*(2 (2018)), 14-17.

Lekishvili, T., & Datuashvili, D. (2019). Regression Models in Estimating Marketable Efficiency of an Assortment of Product. *International Journal of Economics, Commerce and Management, VII*(7).

Lekishvili, T. (2018). Theoretical Foundation of Multi-product Prices Optimization Model (Part I). *Journal of Business*, 27-30.

Lekishvili, T., & Datuashvili, D. (2018). Substantiation of Multi-Product Prices Optimization Model. *Business Engineering, 03*(04), 203-213.