Savi zRvis saerTaSoriso universiteti
kompiuteruli teqnologiebisa da sainJinro saqmis fakulteti

jumhur aqsu

amZravebis terminaluri marTvis axali adapturi meTodis SemuSaveba

sadoqtoro disertaciis avtoreferati

inJineriis mecnierebaTa
(avtomatizacia da marTvis sistemebi)

Tbilisi, 2010

xelmZRvaneli:

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samuSaos zogadi daxasiaTeba

kvlevis Temis aqtualoba.

amZravis sistemebi TamaSoben umniSvnelovanes rols Tanamedrove sawarmoo prowesebSi, amitom maTi dinamikuri procesebis marTvis efeqturi meTodebis SemuSaveba warmoadgens erT-erT yvelaze aqtualur samecniero mimarTulebas.

amZravis marTvis sistemebis SemuSavebisas erT-erT yvelaze mniSvnelovan, samecniero da inJinrul Tvalsazrisit, sakiTxs warmoadgens amZravebis brunvis, siCqaris da aCqarebis marTva. es ukanaskneli miekuTvneba sakmaod rTuli amocanebis klass. misi gadawyvetis arsebuli meTodebi xasiaTdeba didi moculobiTa da sirTuliT, rac iwvevs rTuli programuli da teqnikuri uzrunvelyofis gamoyenebis aucileblobas, rac Tavis mxriv, aZvirebs teqnologiur procesebs mTlianobaSi.

mocemuli amocanebis gadawyvetisas gamoyenebuli meTodebis umravlesoba warmoadgens programuli marTvis optimalur meTodebs (ganrTuli meTodebi ukukavSiris gareSe). aseTebs ganekuTvneba maqsimumis principi, dinamikuri programirebis meTodi, momentebis meTodi da sxva. rogorc aRiniSna, yvela CamoTvlili meTodi programulia, anu iTxovs marTvis kanonis $u(t)$ winaswar gamoTvlas da ar iZleva moZraobis procesSi misi koreqtirebis saSualebas. amavdroulad praqtika iTxovs avtomaturi regulirebis sistemebis (ars) agebas, romlebic ukukavSiris principს iyenebs, rac moZraobis procesSi moZraobis traeqtoriis koreqtirebis saSualebas iZleva.

garda amisa, radgan umetes SemTxvevaSi teqnologiuri procesebi iTxoven moZraobis saboloo stadiaze zusti pozicionirebis uzrunvelyofas, amitom gansakuTrebul aqtualobas iZens swored maTi saboloo mdgomareobebis (terminaluri mdgomareobebis) marTva. amis warmatebiT gadawyveta teqnologiuri procesebis gaumjobesebis saSualebas iZleva. naTqvamidan gamomdinareobs, rom moZravi obieqtebis terminalari (saboloo) mdgomareobebis marTvis amocanebi warmoadgenen aqtualur samecniero da sainJinro sakiTxs da pasuxobs teqnologiebis ganviTarebis Tanamedrove moTxovnebs.

samuSaos miznebi. samuSaos aqtualurobidan gamomdinare sadisertacio kvlevis miznebidan iyo gansazRvruli rogorc:

1- amZravTa sistemebis terminaluri mdgomareobis marTvis axali algoriTmebis SemuSaveba;
2- amZravTa sistemebis terminaluri marTvis kanonis sinTezis SemuSaveba specialuri maTematikuri modelis safuZvelze;
3- miRebuli marTvis kanonis Semowmeba sasimulacio matematikuri uzrunvelyofis paketebis meSveobiT;
4- SemoTavezebuli meTodebis koreqtulobis da efeqturobis verifikacia.

kvlevis obieqti: amZravis sistemebi.

mecnieruli siaxle mdgomareobs SemdegSi:

1. amZravebis sistemebis marTisaTvis SemuSavebulia axali da originaluri terminaluri marTvis meTodi;
2. aRniSnuli meTodi iqna pirvelad dasabuTebuli klasikuri variaciuli meTodebis meSveobiT;

3. zemoTaRniSnulis safuZvelze miRebuli iqna amZravebis sistemebis marTvis martivi da saimedo algoriTmebi, romlis safuZvelze terminaluri marTvis sxvadasxva amocanebi iqna amoxsnili (sxvadasxva sasazRvro pirobebisaTvis);
4. dinamikuri modelirebis meTodis safuZvelze pirvelad iqna miRebuli amZravis pirdapiri marTvis kanoni;
5. miRebulma Sedegebma saSuelebas iZleva miRebuli marTvis kanonis gamoyenebisa amZravebis sistemebis marTvis sxvadasxva amocanebSi.

kvlevis meTodika.

warmodgenil naSromSi gamoyenebulia Semdegi meTodebi:

terminaluri marTvis Teoriis meTodebi, marTvis variaciuli meTodebi, Cveulebrivi diferencialuri gantolebebis meTodebi, maTematikuri modelirebis meTodebi, Matlab Simulink-ze da MatCad-ze programirebis da modelirebis meTodebi.

naSromis praqtikuli mniSvneloba

mdgomareobs imaSi, rom damuSavebuli algoriTmebi SeiZleba warmatebiT gamoyenebul iyos amZravebis marTvis teqnikur sistemebSi, iseTi rogoric aris: samedicino, eleqtrikuli, samxedro da sxva daniSnulebis sistemebi, rasac SeuZlia sagrZnoblad gaaumjobesos teqnologiuri procesebis xarisxi mTlianobaSi.

gamoqveynebuli Sromebi.

samuSaos Temaze gamoqveynebulia 4 naSromi.

naSromis struqtura da moculoba

samuSao moicavs 113 nabeWd gverds, Seicavs 4 Tavs, literaturis CamonaTvals, danarTebs da 43 naxazs.

naSromis Sinaarsi

pirvel TavSi

gadmocemulia sakiTxis mdgomareobis analizi. ganxilulia sasoflo-sameurneo samuSaoebSi robot-manipulatorebis gamoyenebis problemebi. gaanalizebulia agreTve robot-manipulatorebis samuSao organoebis sivrciTi moZraobebis marTvis problemebi, ganxilulia terminaluri marTvis amocanebi. formulirebulia kvlevis miznebi da amocanebi.

meore TavSi

eZRvneba moZravi meqanikuri obieqtebis saboloo mdgomareobis marTvis amocanebis gadawyvetis Teoriul meTodebs.

SemuSavebuli midgomis ZiriTadi idea efuZneba im TvalsaCino faqts, rom moZraobisas moZrav obieqtze moqmedebs ori saxis Zala: mmarTveli da arammarTveli. mmarTveli Zalebis cvlileba agreTve arammarTvel Zalebsac cvlis. moZrav obieqtze moqmedi yvela Zala (arammarTveli + mmarTveli) obieqtis $\dot{V}$ aCqarebas iwvevs. cxadia, rom is advilad eqvemdebareba pirdapir gazomvas, amitom unda daisvas mmarTveli funqciis $\ddot{\gamma}(t)$ aCqarebis saxiT sinTezis amocana, rasac mivyevarT Semdegi variaciuli amocanis amoxsnamde: mocemulia ori wertili $(\gamma_0;\dot{\gamma}_0)$ da $(\gamma_f;\dot{\gamma}_f)$ organzomilebian fazur sivrceSi, saWiroa ganisazRvros fazuri sivrcis iseTi mrudis gantoleba, romelic aerTianebs maT da minimums aniWebs Semdeg funqcionals

$$J_F = \frac{1}{T}\int_0^T f^2(t,\gamma,\dot{\gamma},\alpha(t))dt. \qquad \text{(A)}$$

(A) funqcionali meore rigis warmoebulebis Semcveli tipis funqcionalebs ganekuTvneba, amitom misi Sesabamisi eileris gantoleba SeiZleba Caiweros Semdegi saxiT:

$$\frac{d^2\ddot{\gamma}}{dt^2}=0. \qquad (10)$$

(19)-is amonaxsni mesame rigis polinomia

$$\gamma = C_0 + C_1 t + C_2\frac{t^2}{2} + C_3\frac{t^3}{6}. \qquad (11)$$

sasazRvro pirobebi tolia

$$t=0;\ \gamma=\gamma_0;\ \ \dot{\gamma}=\dot{\gamma}_0, \qquad (12)$$

$$t=T;\ \gamma=\gamma_f;\ \ \dot{\gamma}=\dot{\gamma}_f. \qquad (13)$$

es oTxi piroba sakmarisia imisTvis, rom ganisazRvros oTxi mudmiva C_i (i=0,1,2,3), rac srulad gansazRvravs optimalur traeqtorias.

moTxrobili midgoma sakmaod zogad xasiaTs atarebs, rac saSualebas iZleva gadawyvetili iyos terminaluri marTvis rigi praqtikulad mniSvnelovani amocana. kerZod, igulisxmeba gaqanebis, dayvanisa da daaxloebis amocanebi.

adgilis ekonomiis mizniT mogvyavs mxolod daaxloebis amocanasTan dakavSirebuli Sedegebi, radgan swored isini iyo gamoyenebuli robot-manipulatoris brunvis marTvis praqtikuli algoriTmis SemuSavebisas. aRvniSnoT, rom sadisertacio naSromSi agreTve warmodgenilia gaqanebisa da dayvanis amocanebis sruli amonaxsnebi.

daaxloebis amocanaSi gamoiyeneba oTxive sasazRvro piroba (12) da (13), romlebic saSualebas iZleva pirdapir gamoiTvalos C_i (i=0,1,2,3) koeficientebi mmarTvel funqciaSi.

$$C_0=\gamma_0; \qquad C_1=\dot{\gamma}_0; \qquad C_2=\frac{6}{T^2}(\gamma_f-\gamma_0)-\frac{2}{T}(\dot{\gamma}_f+2\dot{\gamma}_0);$$

$$C_3=\frac{12}{T^3}(\gamma_0-\gamma_f)+\frac{6}{T^2}(\dot{\gamma}_f+\dot{\gamma}_0). \qquad (14)$$

rac saSualebas iZleva miviRoT marTvis sinTezirebuli funqcia

$$\ddot{\gamma}(t) = \left(\frac{6}{T^2}(\gamma_f - \gamma_0) - \frac{2}{T}(2\dot{\gamma}_f + \dot{\gamma}_0)\right) + \left(\frac{12}{T^3}(\gamma_0 - \gamma_f) - \frac{6}{T^2}(\dot{\gamma}_f + \dot{\gamma}_0)\right)t \tag{15}$$

magram es aris ganrTuli (programuli) marTvis kanoni, anu marTvis kanoni uku kavSiris gareSe. aCqarebis pirdapiri gazomvis SesaZleblobis gamoyenebiT marTvis obieqti SeiZleba gadavaqcioT marTvis kanonad uku kavSiriT. am mizniT sakmarisia sawyisi fazuri mdgomareoba mimdinared CaiTvalos, anu davTqvaT $\gamma = \gamma_0$ u $\dot{\gamma} = \dot{\gamma}_0$. amasTan igulisxmeba, rom marTvis obieqti iseTnairad misdevs saboloo wertils γ_f, rom davalebis Sesrulebis dro $T - t = \Delta T$ mudmiv sidided CaiTvalos. maSin vRebulobT mmarTvel funqcias uku kavSiriT marTvisTvis

$$\ddot{\gamma}(t) = \frac{6\gamma_f}{(\Delta T)^2} - \frac{6\gamma}{(\Delta T)^2} - \frac{4\dot{\gamma}}{(\Delta T)} - \frac{2\dot{\gamma}_f}{(\Delta T)}, \tag{16}$$

procesis mocemul mdgomareobaSi gayvanis zogadi amonaxsnis iZulebiTi da gardamavali mdgenelebia

$$\gamma_{fr} = \frac{\Delta T^2}{6}[K_0 - \frac{2}{3}\Delta T K_1 + \frac{5}{9}\Delta T^2 K_2 - \frac{4}{9}\Delta T^3 K_3 + \left(K_1 - \frac{4}{3}\Delta T K_2 + \frac{5}{3}\Delta T^2 K_3\right)t +$$

$$+(K_2 - 2\Delta T K_3)t^2 + K_3 t^3].\ \ \gamma_{tr} = e^{-\frac{2t}{\Delta T}}\left(A\cos\frac{\sqrt{2}}{\Delta T}t + B\sin\frac{\sqrt{2}}{\Delta T}t\right), \tag{17}$$

sadac

$$A = \gamma_{10} - \frac{1}{6}\Delta T^2 K_0 + \frac{1}{9}\Delta T^3 K_1 - \frac{5}{54}\Delta T^4 K^2 + \frac{4}{54}\Delta T^5 K_3;$$

$$B = \sqrt{2}\,\gamma_{10} + \frac{\sqrt{2}}{2}\dot{\gamma}_{10}\Delta T - \frac{\sqrt{2}}{6}\Delta T^2 K_0 + \frac{\sqrt{2}}{36}\Delta T^3 K_1 + \frac{5\sqrt{2}}{270}\Delta T^4 K_2 - \frac{7\sqrt{2}}{108}\Delta T^5 K_3.$$

xazi gavusvaT, rom moyvanil gamosaxulebebSi sawyisi mniSvnelobebi γ_{10} u $\dot{\gamma}_{10}$ ar udris (12)-Si moyvanil sawyis mniSvnelobebs, ris gamoc Cndeba gardamavali procesi (17), romelic drois ganmavlobaSi qreba (amasTan drois mudmiva udris $\frac{\Delta T}{2}$), anu obieqti gamodis iZulebiT traeqtoriaze (27), rasac mivyevarT daaxloebis amocanis srul amoxsnamde.

xSirad terminaluri marTvis praqtikuli amocanebis gadasawyvetad daaxloebis amocanis oTxi sasazRvro piroba (12) arasakmarisia. magaliTad, damuxruWebisas ar kmara saboloo siCqaris nulTan gatoleba. sruli gaCerebisTvis aucilebelia agreTve saboloo aCqarebis nulTan gatoleba. maSasadame, Cndeba aCqarebasTan dakavSirebuli damatebiTi sasazRvro piroba (mexuTe)

$$t=0;\ \gamma=\gamma_0;\ \dot\gamma=\dot\gamma_0,\qquad t=T;\ \gamma=\gamma_f;\ \dot\gamma=\dot\gamma_f;\ \ddot\gamma=\ddot\gamma_f. \tag{18}$$

ar mogvyavs martivi, Tumca sakmaod grZeli gardaqmnebi da pirdapir warmovadgenT mmarTveli funqciisTvis saboloo gamosaxulebas

$$\ddot\gamma(t)=\frac{12}{(T-t)^2}\left(\gamma_f-\gamma\right)-\frac{6}{(T-t)}\left(\dot\gamma_f+\dot\gamma\right). \tag{19}$$

aCqarebis saboloo mniSvnelobas vgulisxmobT $\ddot\gamma_f=0$ saxiT, rac bunebrivia damuxruWebis (gaCerebis) amocanisTvis.

Semdeg moyvanilia robot-manipulatorebis sivrciTi moZraobis marTvis praqtikuli algoriTmebi da am procesebis modelirebis Sedegebi.

ganzogadebuli sivrciTi brunvebis spinoruli warmodgenis safuZvelze SemuSavebuli sivrciTi brunvebis kinematikis spinorulma modelma (Tavi II) da meqanikuri obieqtebis moZraobis saboloo mdgomareobis marTvis Teoriis meTodebma (Tavi III) saSualeba mogvces, Segveqmna robot-manipulatorebis sivrciTi brunvebis saboloo mdgomareobis marTvis martivi meTodebi.

II Tavis Teoriam saSualeba mogvca sivrciTi moZraobis marTvis arsebiTad samganzomilebiani amocana dagveyvana erTganzomilebianze, radgan mbrunavi veqtoris koordinatebi ganisazRvra brunvis erTi, brunvis sibrtyeSi mdebare kuTxis funqciis saxiT. cxadia, rom msgavsi saxis moZraobis Sesabamisi traeqtoriebi sami bunebrivi monakveTisgan

Sedgeba: gaqanebis, Tanabari brunvisa da damuxruWebis, romelTa samarTavad gamoyenebuli iyo III Tavis Sedegebi.

mesame Tavi

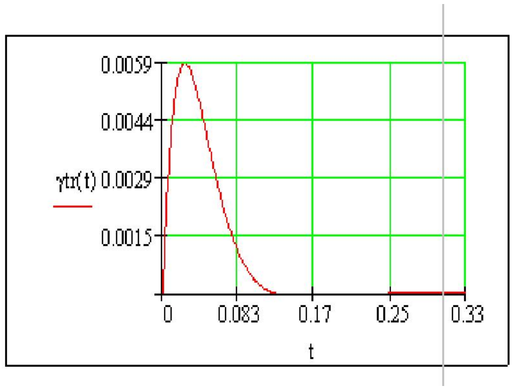
marTvis amocana formulirebuli iyo Semdegnairad. saWiroa brunvis meSveobiT gadaviyvanoT $x(x^1, x^2, x^3)$ koordinatebis mqone marTvis meqanikuri obieqti (mo) (magaliTad, CamWeri an sferuli rgoli) samganzomilebiani sivrcis $y(y^1, y^2, y^3)$ koordinatebis mqone wertilSi.

sawyis etapze brunvis procesi modelirebul iyo MatCAD-is saSua-lebebiT. sawyisi da saboloo veqtorebis saxiT aRebuli iyo x(10,-45,30) da y(1,20,51.225). cxadia, rom maT Soris kuTxe tolia $\gamma_f = ar\cos(\frac{(x,y)}{\|x\|^2}) = 77.65^0$, romelic daiyo sam tol nawilad $\gamma_f/3$=25.880; $2\gamma_f/3$=51.770 da γ_f=77.650, anu, am SemTxvevaSi $\alpha_1 = \frac{1}{3} u \alpha_2 = \frac{2}{3}$. SemdgomSi gamoviyenoT radianebSi gamosaxuli kuTxeebis mniSvnelobebi, amitom $\gamma_f/3$=0.452; $2\gamma_f/3$=0.904 da γ_f=1.355., miviRoT, rom kuTxuri siCqare tolia ω = 1 da brunvis droc agreTve T = 1 wm. davuSvaT agreTve isic, rom samive etapis xangrZlivoba erTnairia, anu rom $T_1 = \frac{T}{3} = 0.333$ wm.

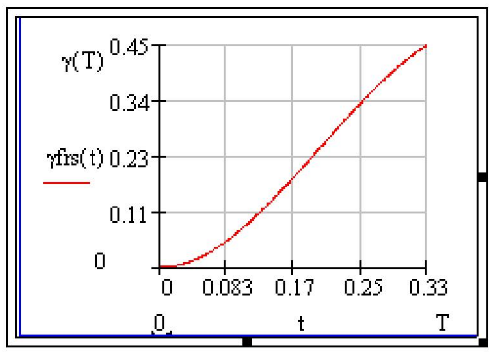
radgan amJamad vixilavT brunvis sawyis etaps, sasazRvro pirobebs Semdegi saxe eqneba

t=0; $\gamma_0 = 0$; $\dot{\gamma}_0 = 0$, t=$_{T1;}$ $\gamma_f = 0.452$; $\dot{\gamma}_f = \omega_f = 1$.

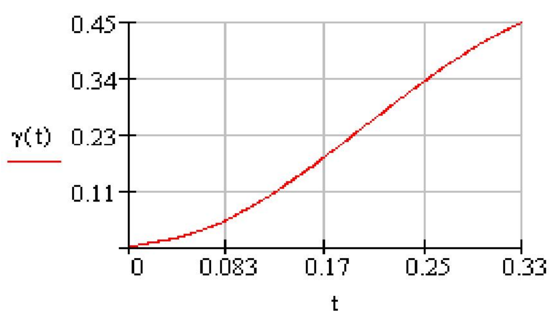
zemoT moyvanili Sedegebis gamoyenebiT miviRebT sur.2-ze moyvanil brunvis sawyisi etapis marTvis dinamikur maxasiaTeblebs.



a)



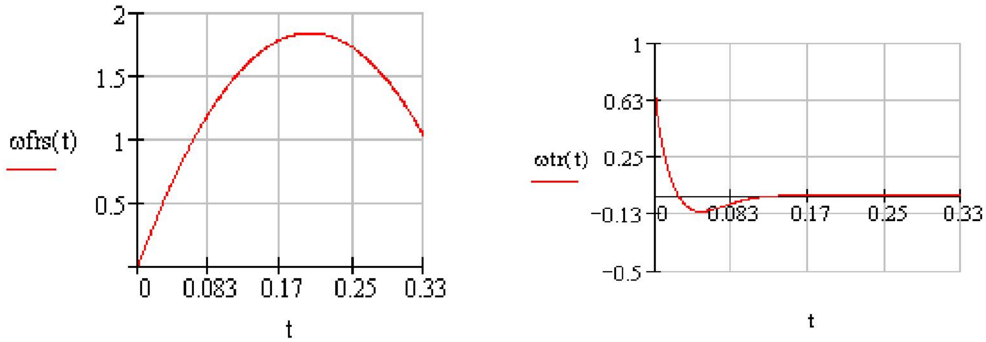
b)



g)

sur.2. moZraobis sawyisi ubani: brunvis kuTxis sididis droze damokidebuleba:

a) iZulebiTi mdgeneli; b) gardamavali mdgeneli; g) fazuri traeqtoria.



a) b)

sur.3. moZraobis sawyisi ubani: kuTxuri siCqaris droze damokidebuleba:

a) iZulebiTi mdgeneli; b) gardamavali mdgeneli;

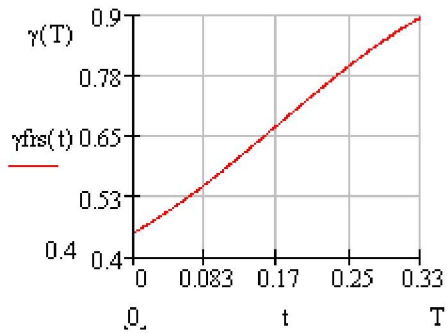
sur. 2b aCvenebs gardamavali procesis arsebobas, Tumca is ori rigiT sustia iZulebiT Semadgenelze (sur. 2a) da sakmarisad swrafad miileva. aseve swrafad miileva kuTxuri siCqaris gardamavali Semadgenelic (sur. 3b), Tumca misi rigi SedarebiTia iZulebiTi Semadgenlis rigTan (sur. 3a). fazur traeqtoriasTan arsebuli susti CaRunvebi (sur. 2g) aris gardamavali procesis Sedegi.

Tanabari brunvis ubanze marTva ar icvleba, anu gamoiyeneba yvela is gantoleba da damokidebuleba, romlebic gamoiyeneboda sawyis ubanze. icvleba mxolod sasazRvro pirobebi

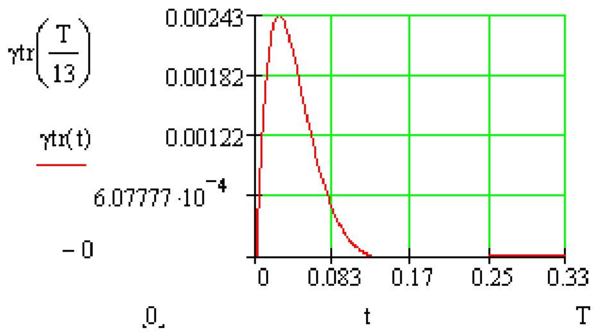
t=0; $\gamma_0 = 0.452$; $\dot{\gamma}_0 = 1$,

t=T1; $\gamma_f = 0.904$; $\dot{\gamma}_f = \omega_f = 1$. (31)

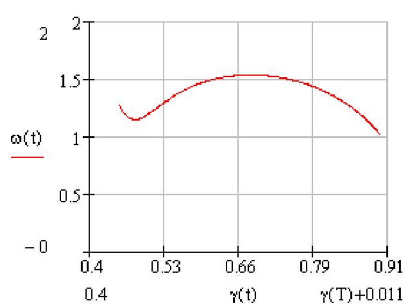
sur. 4-ze naCvenebia marTvis procesis dinamikuri maxasiaTeblebi Tanabari brunvis ubanze:



a)



b)

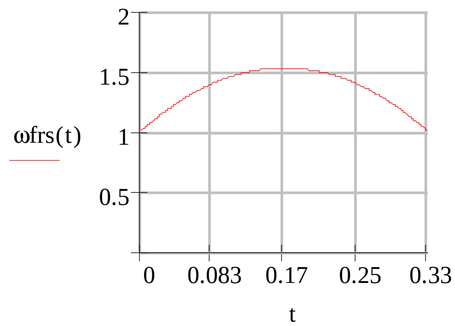


g)

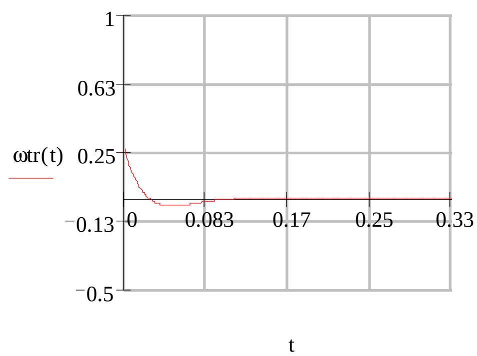
sur. 4. Tanabari brunvis ubani: brunvis kuTxis damokidebuleba droze:

a) iZulebiTi Semadgeneli; b) gardamavali Semadgeneli;

g) fazuri traeqtoria.



a)



b)

sur. 5. Tanabari brunvis ubani: kuTxuri siCqaris damokidebuleba droze:

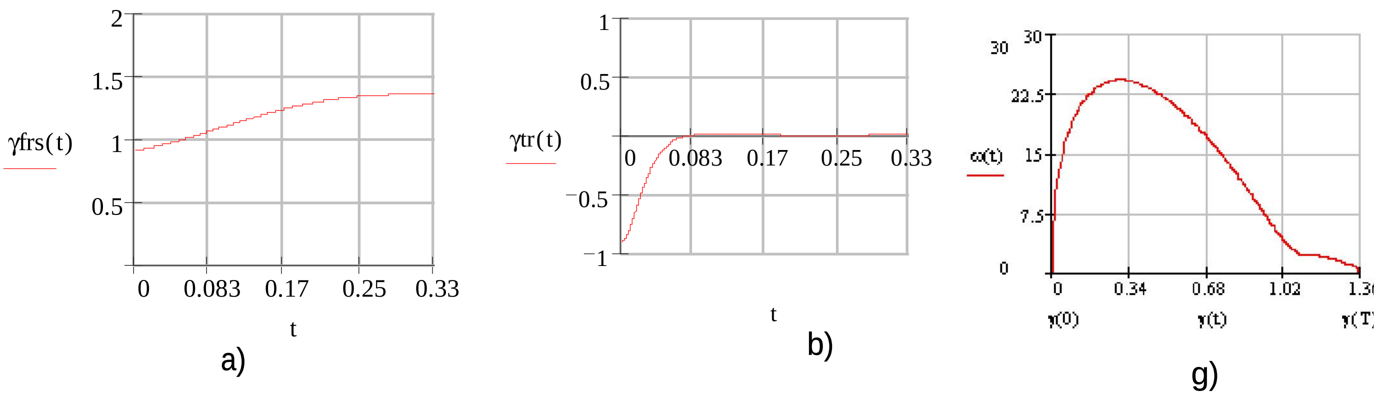
a) iZulebiTi Semadgeneli; b) gardamavali Semadgeneli;

kidev erTxel kargad Cans, rom marTva kargad amuSavebs mo-is sasazRvro pirobebs marTvis periodis boloSi T=0.33 wm. namdvilad gaaCnia mocemuli kuTxuri koordinati $\gamma_f = 0.904$ da siCqare $\dot{\gamma}_f = 1$. adgili aqvs gardamaval process, Tumca gardamavali Semadgeneli kuTxuri koordinatis funqciisTvis umniSvneloa (sur. 3g) maSin, rodesac misi siCqaris funqcia (sur. 5b) Sedarebadia iZulebiTTan (sur. 5a).

sruli gaCerebiT dasrulebuli damuxruWebis procesi iTxovs xuTi pirobis mqone amocanis gamoyenebas, radgan cxadia, rom brunvis boloSi aCqareba nulis toli unda iyos. naTqvamis gaTvaliswinebiT, sasazRvro pirobebi Semdeg saxes iRebs

t=0; $\gamma = 0.904$; $\dot{\gamma} = 1$,

t=T; $\gamma = 1.355$; $\dot{\gamma} = 0$; $\ddot{\gamma} = 0$.

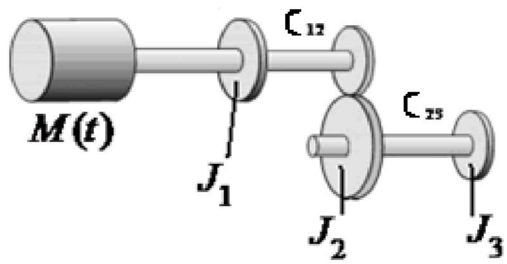


sur. 6. damuxruWebis ubani (gardamavali procesis arseboba): brunvis kuTxis sididis damokidebuleba droze: a) iZulebiTi mdgeneli; b) gardamavali mdgeneli; g) fazuri traeqtoria.

meoTxe TavSi
kontrolis procesebis modelireba

modeli

am TavSi ganxlulia ganvixilavT sami mbrunavi obieqtis mqone sistemas. am modelis ZiriTadi sqema naCvenebia naxazi 3.1-ze.



naxazi 3.1 amZravi sistemis gamosaxuleba

am sistemis maTematikuri modelis Sesaqmnelad saWiroa gamovsaxoT misi dinamika Sesabamisi diferencialuri gantolebebiTa sistemis meSveobiT. kargad nacnobi klasikuri meTodebis gamoyenebiT aRniSnulis dawera rTuli ar aris.

$$
\begin{aligned}
&J_1\ddot{\gamma}_1 + c_{12}\gamma_1 - c_{12}\gamma_2 = M(t) \\
&J_2\ddot{\gamma}_2 - c_{12}\gamma_1 + c_{12}\gamma_2 + c_{23}\gamma_2 - c_{23}\gamma_3 = 0 \qquad (0.1) \\
&J_3\ddot{\gamma}_3 + c_{23}\gamma_3 - c_{23}\gamma_2 = 0
\end{aligned}
$$

sadac

$M(t)$ – Zravis mier generirebuli gare grexviTi momenti ;

J_1 – mqnevara Tvlis, lilvis da pirveli gadacemaTa kolofis inerciis momenti;

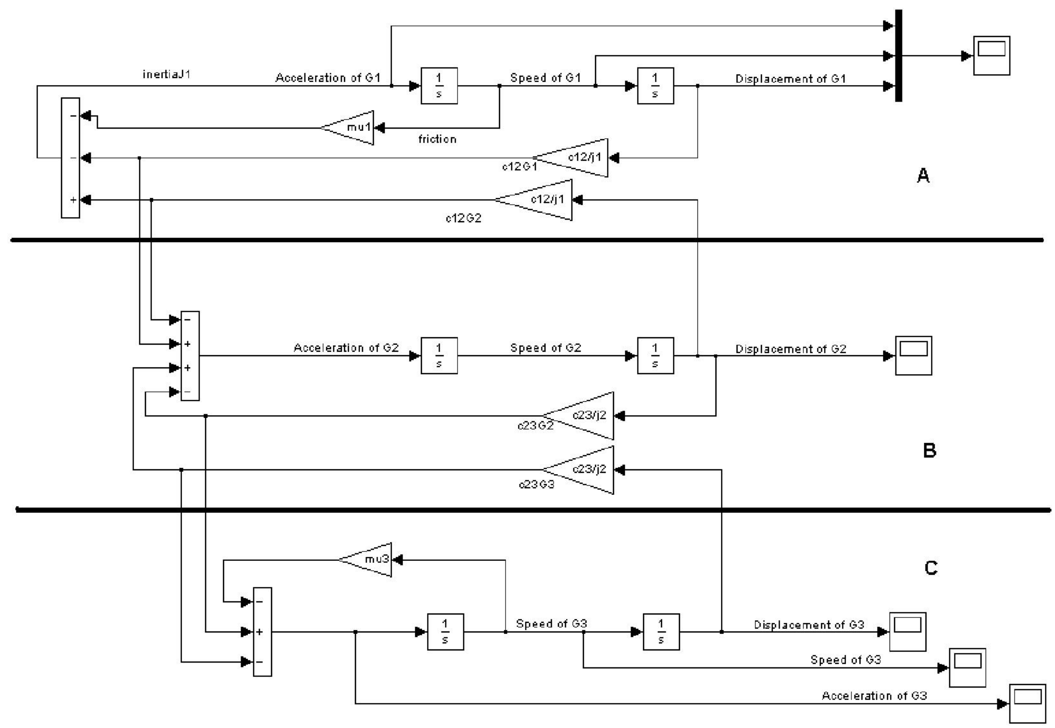
J_2 – sxva gadacemaTa kolofebis seqciebis inerciis momenti

J_3 – saburavis inerciis momenti;

c_{12} – Zravasa da pirvel gadacemis kolofs Soris nawilis grexviTi daWimulobis koeficienti;

c_{23} – sxva gadacemis kolofebsa da saburavs Soris nawilis grexviTi daWimulobis koeficienti.

ganxiluli sistema ar moicavs winaaRmdegobis Zalebs (Cven ganvixilavT sistemis modelს xaxunis Zalis gareSe). cxadia, rom gare impulsis funqcia M(t) uzrunvelyofs sistemis kontrols. Sesabamisad, Cveni pirveli amocanaa ganvsazRvroT M(t)-s funqcia, romelic akmayofilebs aucilebel winaswar gansazRvrul moTxovnebs. azri imaSi mdgomareobs, rom gaCerebis SemTxvevaSi sakmarisi araa miviCnioT, rom terminaluri siCqare nulis tolia: saboloo gaCerebisTvis aucilebelia, rom terminaluri aCqarebac iyos nulis toli.



suraTi 4.1.

A - modelis pirveli gantolebis simulacia, B - modelis meore gantolebis simulacia, C - modelis mesame gantolebis simulacia

modelireba Mathcad-Si

$\gamma 10 := 0.904 \quad \omega 10 := 450$

$\gamma 0 := 0.904 \quad \omega 0 := 450 \qquad \gamma f := 1.355 \qquad \omega f := 0 \quad d\omega f := 0 \qquad T := 7 \qquad dt := \frac{T}{16}$

$S0 := 12\cdot\frac{(\gamma f - \gamma 0)}{T^2} \qquad S0 := S0 - 6\,\frac{(\omega f + \omega 0)}{T} \qquad S0 := S0 + d\omega f$

$S1 := 48\cdot\frac{(\gamma 0 - \gamma f)}{T^3} \qquad S1 := S1 + 18\,\frac{(\omega f + \omega 0)}{T^2} \qquad S1 := S1 - 6\cdot\frac{d\omega f}{T}$

$S2 := 36\cdot\frac{(\gamma f - \gamma 0)}{T^4} \qquad S2 := S2 - 12\,\frac{(\omega f + \omega 0)}{T^3} \qquad S2 := S2 + 6\cdot\frac{d\omega f}{T^2}$

$k_1 := \left(12\,\frac{\gamma 0}{dt^2}\right) + \left(6\,\frac{\omega 0}{dt}\right) + S0 \qquad k_2 := \left(12\,\frac{\omega 0}{dt^2}\right) + \left(6\,\frac{S0}{dt}\right) + S1 \qquad k_3 := \left(6\,\frac{S0}{dt^2}\right) + \left(3\,\frac{S1}{dt}\right) + S2 \qquad k_4 := \left(2\,\frac{S1}{dt^2}\right) + \left(2\,\frac{S2}{dt}\right) \qquad k_5 := \left(\frac{S2}{dt^2}\right)$

$k\gamma := \frac{12}{dt^2} \qquad k\omega := \frac{6}{dt}$

$a4 := \frac{k_5}{k\gamma} \quad a3 := \frac{(k_4 - 4\cdot k\omega\cdot a4)}{k\gamma} \quad a2 := \frac{(k_3 - 3\cdot k\omega\cdot a3 - 12\cdot a4)}{k\gamma} \quad a1 := \frac{(k_2 - 2\cdot k\omega\cdot a2 - 6\cdot a3)}{k\gamma} \quad a0 := \frac{(k_1 - 2\cdot a2 - a1\cdot k\omega)}{k\gamma}$

$t := 0, 0.0001 .. T \qquad \beta := \sqrt{k\gamma - \left(\frac{k\omega}{2}\right)^2}$

$c1 := \gamma 10 - a0 \qquad c2 := \left(\omega 10 - a1 + k\omega\cdot\frac{c1}{2}\right)\cdot\frac{1}{\beta}$

$c1 = -9.437 \times 10^{-15} \qquad c2 = -3.07 \times 10^{-14} \qquad \beta = 3.959 \quad k\omega = 13.714 \quad a0 = 0.904 \quad a1 = 450$

$\gamma tr(t) := e^{\frac{-k\omega\cdot t}{2}}\cdot(c1\cdot\cos(\beta\cdot t) + c2\cdot\sin(\beta\cdot t))$

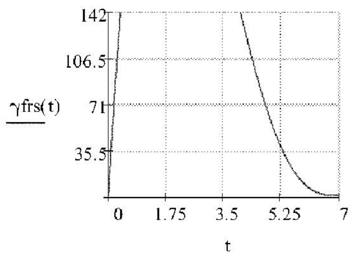
$\omega tr(t) := e^{\frac{-k\omega\cdot t}{2}}\cdot\left(\frac{-k\omega}{2}\right)\cdot(c1\cdot\cos(\beta\cdot t) + c2\cdot\sin(\beta\cdot t)) + e^{\frac{-k\omega\cdot t}{2}}\,(-c1\cdot\beta\cdot\sin(\beta\cdot t) + c2\cdot\beta\cdot\cos(\beta\cdot t))$

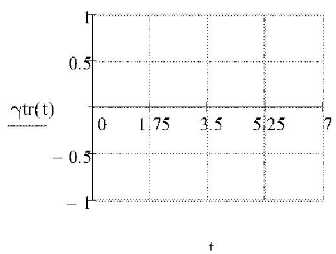
$\gamma frs(t) := \left(a0 + a1\cdot t + a2\cdot t^2 + a3\cdot t^3 + a4\cdot t^4\right)$

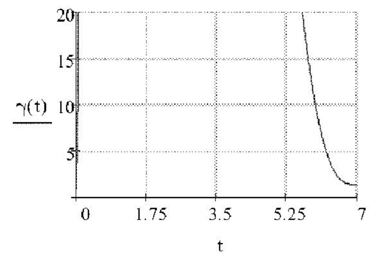
$\omega frs(t) := a1 + 2\cdot a2\cdot t + 3\cdot a3\cdot t^2 + 4\cdot a4\cdot t^3 \qquad \omega tr(0) = -5.684 \times 10^{-14}$

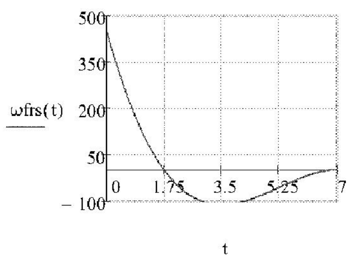
$\gamma(t) := (\gamma frs(t) + \gamma tr(t)) \qquad \omega(t) := \omega tr(t) + \omega frs(t) \qquad \gamma frs(0) = 0.904 \qquad pp := \omega 10 - a1$

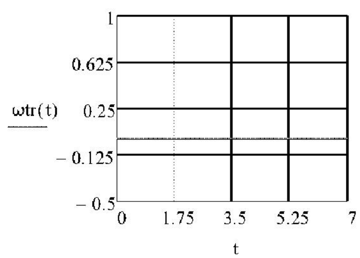
$dt = 0.438 \qquad a(t) := \frac{d}{dt}\,\omega(t) \qquad \gamma tr(0) = -9.437 \times 10^{-15} \qquad pp = -5.684 \times 10^{-14}$

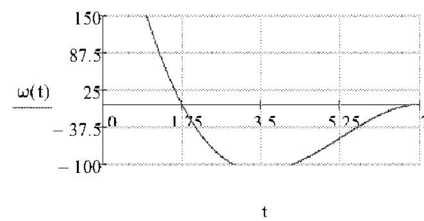


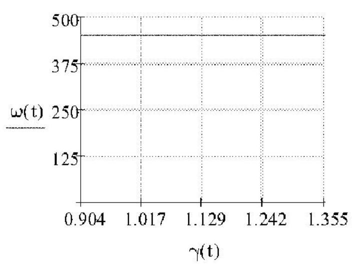


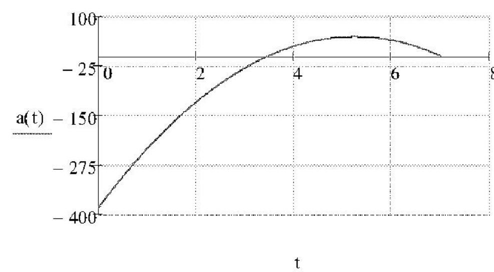












$c12 := 0.5 \qquad c23 := 0.6 \qquad j1 := 1.54\cdot 10^{-6} \qquad j2 := 0.00001 \quad j3 := 8.92\cdot 10^{-3}$

$\delta := \dfrac{k\omega}{2}$

$sm(t) := a0 + a1\cdot t + a2\cdot t^2 + a3\cdot t^3 + a4\cdot t^4$

$ds(t) := a1 + 2\cdot a2\cdot t + 3\cdot a3\cdot t^2 + 4\cdot a4\cdot t^3$

$dds(t) := 2\cdot a2 + 6\cdot a3\cdot t + 12\cdot a4\cdot t^2$

$\mu_1 := 0$

$k(t) := e^{-\delta\cdot t}\cdot(c1\cdot\cos(\beta\cdot t) + c2\cdot\sin(\beta\cdot t))$

$\mu_2 := 0.1$

$p(t) := e^{-\delta\cdot t}(-c1\cdot\sin(\beta\cdot t) + c2\cdot\cos(\beta\cdot t))$

$\mu_3 := 0.9$

$$dg3(t) := -\delta \cdot k(t) + \beta \cdot p(t) + ds(t)$$

$$ddg3(t) := k(t)\cdot\left(\delta^2 - \beta^2\right) - p(t)(2\delta\beta) + dds(t)$$

$$m1(t) := \left[j1\cdot\frac{(j2 + j3)}{c12} + j3\cdot\frac{(j1 + j2)}{c23}\right]\cdot\left[\left(\delta^4 - 6\delta^2\cdot\beta^2 + \beta^4\right)\cdot k(t) + \left(4\cdot\delta\cdot\beta^3 - 4\delta^3\cdot\beta\right)\cdot p(t) + 24\cdot a4\right]$$

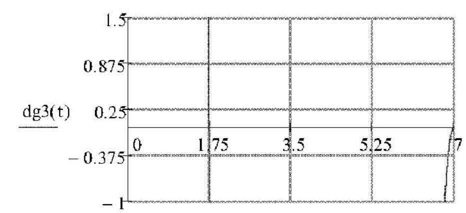
$$m2(t) := \left(j1\cdot j2\cdot\frac{j3}{c12\cdot c23}\right)\cdot\left[\left(\delta^6 - 15\cdot\delta^4\beta^2 + 15\cdot\delta^2\cdot\beta^4 - \beta^6\right)\cdot k(t) + \left(20\cdot\delta^3\cdot\beta^3 - 6\cdot\delta^5\cdot\beta - 6\cdot\delta\cdot\beta^5\right)\right]$$

$$z := \left(1 + \frac{c23}{c12} - c23\right)$$

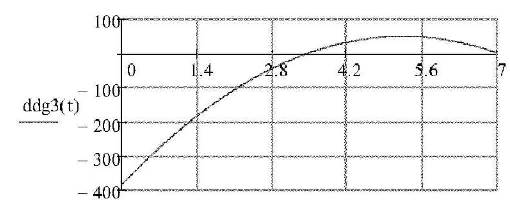
$$m3(t) := (z\cdot j1 + j2 + j3)\cdot\left[\left(\delta^2 - \beta^2\right)\cdot k(t) + (-2\cdot\delta\cdot\beta)\cdot p(t) + \left(12\cdot a4\cdot t^2 + 6\cdot a3\cdot t + 2\cdot a2\right)\right] + (c23 - c12\cdot c23)\cdot(k(t) + sm(t))$$

$$M(t) := m1(t) + m2(t) + m3(t)$$

$$y0 := \begin{pmatrix} 0.904 \\ 450 \\ 0.904 \\ 450 \\ 0.904 \\ 450 \end{pmatrix}$$



$$D(t,y) := \begin{bmatrix} y_2 \\ \dfrac{\left[M(t) + c12\cdot(y_3 - y_1) - y_2\cdot\mu_1\right]}{j1} \\ y_4 \\ \dfrac{\left[c23\cdot(y_5 - y_3) + c12\cdot(y_1 - y_3) - y_4\,\mu_2\right]}{j2} \\ y_6 \\ \dfrac{\left[c23\cdot(y_3 - y_5) - y_6\cdot\mu_3\right]}{j3} \end{bmatrix}$$

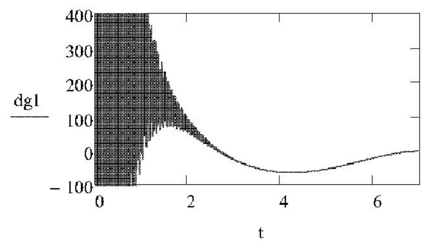


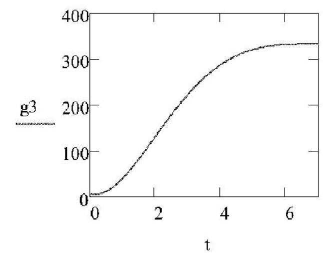
$$yn := Rkadapt(y0, 0, T, 1000, D)$$

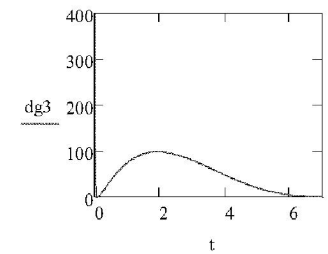
	1	2	3	4	5	6	7	8
1	0	0.904	450	0.904	450	0.904	450	
2	$7 \cdot 10^{-3}$	-8.021	$2.778 \cdot 10^{3}$	0.781	-31.203	3.162	221.589	
3	0.014	-2.118	$-3.193 \cdot 10^{3}$	0.872	6.99	4.271	108.323	
4	0.021	4.985	$2.372 \cdot 10^{3}$	0.883	42.834	4.81	52.187	
5	0.028	-3.496	937.409	1.084	0.607	5.066	24.424	
6	0.035	7.347	$-2.543 \cdot 10^{3}$	1.373	54.02	5.184	10.724	
7	0.042	8.287	$3.391 \cdot 10^{3}$	1.578	53.824	5.233	4.033	
8	0.049	3.596	-944.144	1.995	28.108	5.248	0.84	
9	0.056	15.925	$-1.019 \cdot 10^{3}$	2.39	85.437	5.248	-0.625	
10	0.063	11.683	$3.282 \cdot 10^{3}$	2.769	57.875	5.242	-1.194	
11	0.07	12.668	$-2.223 \cdot 10^{3}$	3.341	59.213	5.233	...	

$yn =$ (matrix above)

$t := yn^{\langle 1 \rangle}$ $\quad g1 := yn^{\langle 2 \rangle}$ $\quad dg1 := yn^{\langle 3 \rangle}$ $\quad g2 := yn^{\langle 4 \rangle}$ $\quad dg2 := yn^{\langle 5 \rangle}$ $\quad g3 := yn^{\langle 6 \rangle}$ $\quad dg3 := yn^{\langle 7 \rangle}$







$$ddg1a(t) := k(t)\cdot\left[\left(j2\cdot\frac{j3}{c12\cdot c23}\right)\cdot\left(\delta^6 - 15\cdot\delta^4\beta^2 + 15\cdot\delta^2\cdot\beta^4 - \beta^6\right) + \left[\left[\frac{(j2+j3)}{c12} + \frac{j3}{c23}\right]\cdot\left(\delta^4 - 6\delta^2\cdot\beta^2 + \beta^4\right)\right] + \left(\delta^2 - \beta^2\right)\cdot z\right]$$

$$ddg1b(t) := p(t)\cdot\left[\left(j2\cdot\frac{j3}{c12\cdot c23}\right)\cdot\left(20\cdot\delta^3\cdot\beta^3 - 6\cdot\delta^5\cdot\beta - 6\cdot\delta\cdot\beta^5\right) + \left[\frac{(j2+j3)}{c12} + \frac{j3}{c23}\right]\cdot\left(4\cdot\delta\cdot\beta^3 - 4\delta^3\cdot\beta\right) + (-2\cdot\delta\cdot\beta)\cdot z\right]$$

$$ddg1c(t) := (24\cdot a4)\cdot\left[\frac{(j2+j3)}{c12} + \frac{j3}{c23}\right] + \left(12\cdot a4\cdot t^2 + 6\cdot a3\cdot t + 2\cdot a2\right)\cdot z$$

$$ddg1(t) := ddg1a(t) + ddg1b(t) + ddg1c(t)$$

Given

$$\frac{d}{dt}y0(t) = y1(t) \qquad y0(0) = 0.904$$

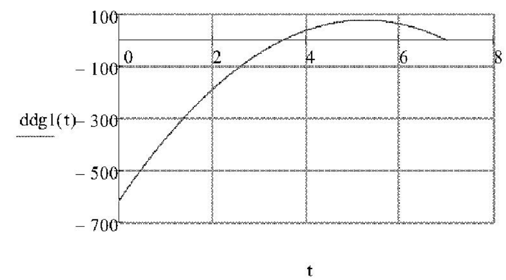
$$\frac{d}{dt}y1(t) = \frac{\left[M(t) + c12\cdot(y2(t) - y0(t)) - y1(t)\cdot\mu_1\right]}{j1} \qquad y1(0) = 450$$

$$\frac{d}{dt}y2(t) = y3(t) \qquad y2(0) = 0.904$$

$$\frac{d}{dt}y3(t) = \frac{\left[c23\cdot(y4(t) - y2(t)) + c12\cdot(y0(t) - y2(t)) - y3(t)\,\mu_2\right]}{j2} \qquad y3(0) = 450$$

$$\frac{d}{dt}y4(t) = y5(t) \qquad y4(0) = 0.904$$

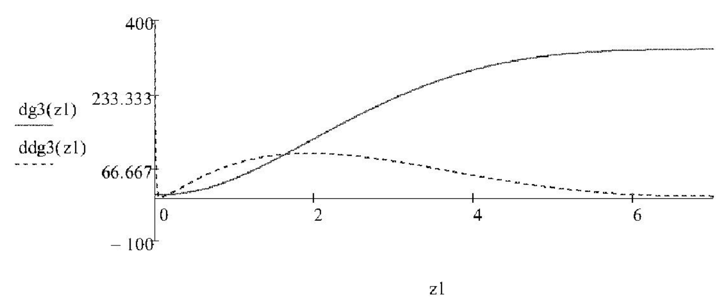
$$\frac{d}{dt}y5(t) = \frac{\left[c23\cdot(y2(t) - y4(t)) - y5(t)\cdot\mu_3\right]}{j3} \qquad y5(0) = 450$$



$$T1 := 7$$

$$\begin{pmatrix} dg1 \\ ddg1 \\ dg2 \\ ddg2 \\ dg3 \\ ddg3 \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} y0 \\ y1 \\ y2 \\ y3 \\ y4 \\ y5 \end{pmatrix}, t, T1, 1\times 10^3\right]$$

$$z1 := 0, \frac{T1}{1\times 10^3} .. T1$$



daskvnebi

1. amZravebis sistemebis marTisaTvis SemuSavebulia axali da origanuluri terminaluri marTvis meTodi;
2. aRniSnuli meTodi iqna pirvelad dasabuTebuli klasikuri variaciuli meTodebis meSveobiT;
3. zemoTaRniSnulis safuZvelze miRebuli iqna amZravebis sistemebis marTvis martivi da saimedo algoriTmebi, romlis safuZvelze terminaluri marTvis sxvadasxva amocanebi iqna amoxsnili (sxvadasxva sasazRvro pirobebisaTvis);
4. dinamikuri modelirebis meTodis safuZvelze pirvelad iqna miRebuli amZravis pirdapiri marTvis kanoni;
5. miRebulma Sedegebma saSuelebas iZleva miRebuli marTvis kanonis gamoyenebisa amZravebis sistemebis marTvis sxvadasxva amocanebSi.

gamoqveynebuli naSromTa sia

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