INTERNATIONAL BLACK SEA UNIVERSITY
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Developing Nonparametric Method For Improving Spectral Resolution of Time Series Spectra Estimation

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Extended Abstract of Doctoral Dissertation of Engineering in Informatics

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Introduction

Modeling, Analysis and forecasting of the time series is one of the most crucial task, because it has huge amount of applications in all field of science, engineering, economics and finance, in all part, where our goal is and we are going to analyze experimental data and make decision based on what result we got and what is required. There exist different type of random processes (having trend, stationary, ergodic, purely random, etc.), but analyses of processes that contain periodical components are also very important in a practice, especially for identification and forecasting in economics and financial sphere. (Bartlett, 1987), (Box & Jenkins, 1976), (Brillinger, 1975), (Brockwell & Davis, 2009), (Chatfield, 1996), (Granger, 1964), (Hamilton, 1994)

Spectral estimation and modeling is especially interesting when we have such time series, which contains periodic components and plus white noise and amount of periodic components is finite

$$x(t) = \sum_{j=1}^{m} A_j e^{2\pi i f_j t} + w(t), \tag{1}$$

In this model we have following parameters A_j- is magnitude of component j

f_j-frequency of deterministic periodic component j

m-number of periodic components

w (t)-is white noise which is independent from periodic components

In discrete form same model will have following form

$$x[n] = \sum_{j=1}^{m} A_j e^{2\pi i f_j n T_s} + w(nT_s), \tag{2}$$

Again in this model variable n- is an index of current sample;

A_j-magnitude of j components

f_j-its frequency

T_S_time interval between two adjacent moment of time (f_S=1/T_S_sampling rate).

w (nT_s) - independent of the periodic components white noise.

While analyzing time series of model (2), important task is to estimated parameters from given model or to estimate the values of m, A_j and f_j ,which is the problem of detection invisible periodicities. From a practical point of view, it has numerous applications in important areas of science and technology: for instance detection of of resonance phenomena in the operation of technical facilities, detection signals in radar technologies and so on.

Main complexity of spectral estimation of time series of type (2) is that stochastic processes which is represented by such time series, are not stationary because of it contains deterministic periodic components. Existence of such components will cause divergence of Fourier transform of corresponding nonsingular integral. Such problems occurs for most of stationary processes, which corresponds to this fact that in such case it is used not Fourier transform of stationary process itself, but Fourier transform of correlation function of stationary time series.Therefore, for analysis of such type of processes, it is considered not their spectra, but their power spectrum., which represent Fourier transform of their correlation function. If process is stationary, during the time shifting, its correlation function is decreasing more rapidly, which is enough for convergence Fourier transform of corresponding correlation function, while for processes itself it can be divergent[1].,The last circumstance corresponds random nature of stationary stochastic processes, during increasing of time shifting, there is decreasing correlation at corresponding shifting time intervals between values of processes, this is represented in rapid decreasing of correlation function and convergence of nonsingular Fourier transform , As a result, in theorem of stationary processes, there exist central Wiener–Khinchin theorem, which determines conditions for existence of power spectrum, It should be noted that, for most of practical tasks, power spectra is more interesting, then spectra of stationary processes itself (Bloomfield, 2000), (Pisarenko, 1973), (Box & Jenkins, 1994), (Bracewell, 2000)

Conditions for existence power spectra determined by Wiener–Khinchin theorem is not satisfied for such signal, which contains deterministic periodic components, for instance let us consider following simplest harmonic signal

$$x(t) = A_1 \sin(\omega_0 t + \varphi) ,$$

Where parameters A_1, ω, φ – are nonrandom real numbers

It is well known, that correlation function of this signal will have following form

$$x(t) = \frac{A_1^2}{2} \cos \omega_0 \tau ,$$

where, τ – is time shift

It is obvious that this is also periodic function and its Fourier form or power spectra represents sum of Dirac-Delta functions

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[1].This property of correlation function can be used as base for determination stationarity more broadly

$$s(\omega) = \frac{A_1^2}{2}\left[\delta(\omega - \omega_0) + \delta(\omega - \omega_0)\right].$$

So, for signals, that contain periodic deterministic components, Fourier integral in terms of usual function class, does not exist, besides this, for such processes, observation duration is always finite, which also causes some kind of distortion in estimation their power spectrum (so called windows problems)

For time series, which represent a discrete sample from processes (signals), it is possible to calculate Fourier image using discrete Fourier Transform, because time series in general is n-dimensional vector, and transform operator is-unitary matrix with rank which means that there does not occurs naturally problem of convergence. .Discrete Fourier transform of time series containing deterministic periodic components we are calling Pseudospectrum, as we want to underlain principal difference from spectrum of stationary processes(detailed explanation of Pseudospectrum is given in paragraph 2.2) (Box & Jenkins, 1994), (Bracewell, 2000)

Pseudospectrum estimation methods are based on analysis of singular values and singular vectors of correlation matrix. There one of important task is resolution problem of periodic components that are close in frequency so called Pseudospectrum resolution. The point is that existed methods for Pseudospectrum estimation ((Multiple Signal Classification (MUSIC), Eigen Values Method and etc) are characterized with low resolution ability, because correlation matrices of time series, which are containing periodic components and that periodic components are closed to each other in frequency, are ill conditioned, this situation does not give us to possibility to separate components Reliably and therefore effectiveness of these methods is decreased significantly and their applicability in separating periodic deterministic components in real practical important tasks are very low. (Laning & Battin, 1976), (Kay, 1993), (Hannan, 1970)

Problem Statement

Task of analysis of time series that consists of sum of periodic deterministic components with additive white noise is a very important in different field of science: in Radar Technologies, in finance, in various engineering problems, etc. Special interest during the analysis of sinusoidal time series is devoted to the problem of resolution and resolving capacity. The problem is that modern existing classical methods for the estimation of pseudospectrum in majority of cases are not able to identify hidden periodic components when their frequencies and amplitudes are close to each other. Therefore our objective is to develop new method for increasing resolving capacity of time series pseudospectra estimation. This method should be applicable to all existing classical methods of both spectra and pseudospectra estimation.

Actuality of topic

Actuality of resolution problem or task of hidden periodicities is very important in modern science. Starting from Radars in army continue to economical process, it should be mentioned that during of processes which occurs in stock markets, many of them are characterized by periodical properties, also in vibration, according to actuality of task, correct estimation of hidden periodicities and successfully resolving of deterministic components from each other is a very crucial factor in designing and modeling of process that is managed by such models.

Novelty of the investigation

- ✓ Has been developed new method for estimation Pseudospectrum of time series consist of periodic deterministic components, that significantly increase resolving capacity
- ✓ Has been Proposed new approach of time series approximation, based on low rank tensor approximation
- ✓ Has been Proposed new iterated method for singular value decomposition of data matrix
- ✓ Has been introduced terms of order of low rank approximation and time of singular sweeping
- ✓ Has been shown that left and right singular vectors of data matrix and original time series have equal pseudospectral structure
- ✓ Has been shown that by concatenation of singular vectors, we can increase time of singular sweeping, which at the same time increase statistical stability and resolution of pseudospectral estimation
- ✓ Based on examples considered in the thesis, Has been shown practical and statistical reliability of given result
- ✓ Based on practical examples made by special programming software, comparative analysis of newly proposed method and existed methods has been done , which has been provided effectiveness of new method

Practical &Theoretical value of research

As far as estimation of pseudospectrum of nonstationary time series and developing new method for solving problem of separation periodic components from this pseudospectrum gives possibility to significantly increase effectiveness of solution of many practical tasks

Structure of Thesis

Thesis consist of introduction, literature review, theoretical part, practical part, conclusion, appendix, 43 figures, 36 tables and 40 references.

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Basic content of research

Chapter 1

In chapter 1, critical analysis of modern classical spectrum estimation methods are done. Critical analysis showed that despite of popularity of those methods, they are characterized by low resolution abilities because of several reason: first they generally assume that process is stationary (for instance Autoregressive models), also ill-conditioness of correlation matrix (in case of Eigenvalue analysis methods) and direct robust estimation power spectrum from given data (Periodogram/correlogram) and finally nonlinear regression estimation(Prony method for analyzing damped sinusoidal models) significantly reduces resolution ability of those methods and also increases statistical instability. On the base of critical analysis of the state of the problem, objectives of the research have been determined

Objectives of the research

1. To develope new method for estimation Pseudospectrum of time series consist of periodic deterministic components, that significantly increase resolving capacity
2. To Propose new approach of time series approximation, based on low rank tensor approximation
3. To Propose new iterated method for singular value decomposition of data matrix
4. To Introduce terms of order of low rank approximation and time of singular sweeping
5. To show that left and right singular vectors of data matrix and original time series have equal pseudospectral structure
6. To show that by concatenation of singular vectors, we can increase time of singular sweeping, which at the same time increase statistical stability and resolution of pseudospectral estimation
7. Based on examples considered in the thesis, to show practical and statistical reliability of given result

8. Based on practical examples made by special programming software, to do comparative analysis of newly proposed method and existed methods, which will provide effectiveness of new method

Chapter 2 Theoretical part

Resolving Capacity

Resoling capacity (also resolution capacity) represents the most important quality characteristic (index) of any classical method of spectral estimation, under this terminology we mean the ability to resolve (to separate) spectral response of two sinusoidal signals that are close in frequency and magnitude (Milnikov, 2014), (Datuashvili, Mert, & Milnikov, 2014), (Milnikov, 2013). It is generally assumed that the frequency separation of the two sine waves cannot be less than the equivalent bandwidth B_e of the spectrum's window (Marple, 1987). Since for time series, that consist by sum of deterministic periodic components, for such signals $B_e T_o \approx 1$, where T_0 is the time of observation of the time series in seconds (total observation time), then the resolution is assumed to be approximately equal to an inverse value of $T_{0.}$ When we have time series that contains stochastic part(random components),then the different estimation criterion of the estimation of the spectral resolution is used in this case: the triple product $QT_e B_e$, where Q – the statistical quality index, defined as the ratio of the variance of the power spectral density (PSD) to the square of the expectation of this estimate. In fact it is - an inverted signal / noise ratio (SNR - Signal / Noise Ratio), which directly defines and is directly connected to the statistical stability of spectral estimation.Acoording to Heisenberg uncertainty, it is already known, that for a given time-series (which time interval of its registration is equal $T_{o),}$it is impossible that estimations have simultaneously a high resolution (small values of the Bs) and high statistical stability - low values of Q (Marple, 1987),note also resolution will not be increased if we increase number of samples by changing(increasing or decreasing) the sampling frequency at a constant time interval, as the resolution ability depends only on the length of the time interval of the data record(total observation time)and not on number of samples.

Simultaneously increasing of spectral resolution and also increasing of statistical stability is the main characteristic of the method that has been developed in this thesis. From one side, it would seem that it contradicts to the fundamental principle mentioned above, but the point is that the Propositions 1 and 2 for deterministic and Proposals 3 and 4 for the random series which will be introduced in the later part of thesis, allow: 1. increasing of the observation time T_0 and 2.

Reducing the value of Q that is, increasing the statistical stability of spectral estimation (Sturrock, Scargle, Walther, & Wheatland, 2005)

Definition of equivalency of pseudospectral structure

Number of peaks m that are fallen within deterministic area I_d and corresponding vector of frequencies f_i (i=1,2,…,m) are important characteristics of the pseudospectra, and they allow introducing notion of **Pseudospectral Structure**.

Let's call a vector of frequencies f_i of a Time Series, such that $P_i = p(f_i) \in I_d \; (i = 1,2,\ldots,m)$, where I_d is deterministic area, **Pseudospectral Structure (PS)** of this Time Series.

Finally definition of Pseudospectral Structure as given above naturally enables us to introduce the notion of the equivalence of the two PS. Let two time series have two deterministic areas I_{d_1} and I_{d_2}, also let $P_{i1} \in I_{d_1} \; (i = 1,2,\ldots,m_1)$ and $P_{i2} \in I_{d_2} \; (i = 1,2,\ldots,m_2)$ be vectors of peaks (deterministic components) belonged to deterministic areas I_{d_1} and I_{d_2} respectively. (Milnikov, 2014)

Definition2. Two time series have the equivalent PS if:

$m_1 = m_2 = m$;

$p^{-1}(P_{i1}) = p^{-1}(P_{i2}), \quad i = 1,2,\ldots,m.$

The first criteria checks whether the comparing psseudospectra have equal number of deterministic components, while second criteria checks – equality of frequencies of the deterministic components (frequencies are defined by means of inverse mappings p^{-1}). Note that the Definition 2 requires analyzing of only stochastic areas I_d, that is we assume that the equivalency of two PS completely determined by their deterministic components and does not depend on random background.

Pseudospectral structure of singular vectors

Time series with periodic component without noise

In this section we are considering sample represented by x[1], x[2],…,x[N] ,where N is the length of the time series. Its data matrix is defined as follows

$$X_d = \begin{vmatrix} x[1] & x[2] & \ldots & x[p] \\ x[2] & x[3] & \ldots & x[p+1] \\ \ldots & \ldots & \ldots & \ldots \\ x[N-p] & x[N-p+1] & \ldots & x[N] \end{vmatrix}.$$

X_d it is – rectangular $p \times l$ matrix, where 0<p<N and l=N-p.

Using low rank tensorial approximation, we can decompose this matrix as

$$X_d = \sum_{i=1}^{q} \lambda_i (u_i \otimes v_i)$$

Where

where $\otimes$ -is the sign of the tensorial product;

q- number of approximation's components ($q \leq r$)[2];

r ($r \leq \min(p,l)$) –is the rank of X_d;

λ_i - are the singular values of X_d;

u_i and v_i – represents left and right singular vectors of X_d, respectively.

Complete theoretical aspects and proof related to this approach is given in Thesis itself, but here we would like to show two important Proposition related to singular vectors of given matrix

Proposition 1. All 2m singular vectors corresponding to nonzero singular values (left and right) of the data matrix X_d, constructed according to the time series (2.18), are linear combinations of the observations of its periodic component.

Proposition 2. All 2m singular vectors corresponding to nonzero singular values (left and right) of the data matrix X_d, constructed according to the time series of the form (2.18), have the same pseudospectra, and their pseudospectral structure is identical to the pseudospectral structure of the original series.

Time series with periodic components with additive noise

When time series is represented by sum of deterministic periodic components and white noise, all the elements of the sampled data will contain noise component, that means that the singular vectors given by the expansion (2.28) will not be composed only by deterministic parts, but be composed of deterministic and noise components. Rank of the data matrix X_d is equal to minimum number between (p,l) and will be more than m, which is one of the 2m +1 (m magnitudes and frequencies) subjects to be estimated. As we have proved in Proposition 1, deterministic components will be present only among the first m terms of the matrix singular values (Datuashvili, Mert, & Milnikov, 2014), (Milnikov, 2014), (Milnikov, 2013)

[2] if q=r, we have accurate representation of X_d by means of decomposition (9).

$$X_d = \sum_{i=1}^{m} \mu_i (u_i \otimes v_i) + \sum_{i=m+1}^{\min(p,l)} \mu_i (u_i \otimes v_i) =$$
$$= \sum_{i=1}^{m} (\lambda_i + \varepsilon_i)(u_i \otimes v_i) + \sum_{i=m+1}^{\min(p,l)} \mu_i (u_i \otimes v_i) \quad (2.28)$$

Important results and propositions that were developed based on such time series, is given below

Proposition 3. Principal singular vectors (left and right) of the data matrix X_d, constructed according to a time series of the form (2.18) are linear combinations of the time series deterministic periodic and random (noise) components. The remaining singular vectors contain only noise components. (Milnikov, 2014)

Proposition 4. Principal singular vectors (right and left) of the data matrix X_d constructed according to the time series of the form (2.18) have the same pseudospectra, and their pseudospectral structure is identical to the pseudospectral structure of the original series. The remaining singular vectors have pseudospectral structure of white noise, different from the original series pseudospectral structures and main singular values.

Proposition 5.. Principal singular vectors (left and right) of the data matrix Xd , constructed according to a time series of the form (2.18) are linear combinations of the time series deterministic periodic and random (noise) components. The remaining singular vectors contain only linear combination of only noisy components.

Based on theorems and propositions that were presented in this paragraph, we have proofed that pseudospectral structure of singular vectors of data matrix are equivalent of pseudospectral structure of original time series , as well as principal singular vectors(left and right) represent as a linear combination of deterministic periodic components.

Chapter 3 Numerical Examples of Spectra Estimation

Here we introduce a criterion for estimating the separation of the two peaks of a pseudospectrum. It is assumed that the two peaks corresponding to two periodic deterministic components are separated, if a "dip" notch between them not less than 3 db, therefore as a criterion of separation of two peaks one should use the following coefficient of seperability (Datuashvili, Mert, & Milnikov, 2014)

$$S(f)=\frac{P(f_j)}{P(f_i)}\geq 10^{0.3}\ (i\neq j),\qquad (3.1)$$

Where $P(f_j)$ - the value of a smaller peak (between two comparable ones) at the frequency f_j;

$P(f_i)$ - Value of notch at frequency f_i.

In this part due to limited amount of volume, we are going to show just one practical example of comparatively analysis between newly developed method and existing classical methods of Pseudospectrum estimation

Comparative analysis of Estimation spectral resolution methods for periodic deterministic signal with high intensity of noise

Again our model of interest is sum of periodic components and noise

$x(t)=A_1\sin(2\pi f_1 t)+A_2\cos(2\pi f_2 t)+w(t)$

Second additive component is with high intensity, in our case noise is 4db with respect to periodic deterministic components, sample size is again $N=294$,sampling frequency was taken as $f_s=100Hz$,sample period therefore equal to $t_s=\frac{1}{f_s}=0.01$ and total time of observation is equal to $T_0=N*t_s=2.94\ sec$, resolution frequency is $f_r=\frac{1}{T_0}=0.34Hz$,amplitudes are chose as $A_1=40, A_2=30\ units.$

Frequencies difference greater than resolution frequency in case of noisy signal

Let us consider same model with parameters, but with frequencies$f_1=10Hz, f_2=10.5Hz$, which salsifies condition that difference between frequencies $\Delta f=f_2-f_1=10.5-10>f_r$,before start analysis of pseudospectrum estimation methods and their resolution frequencies, let us see how graph of this noisy signal looks like

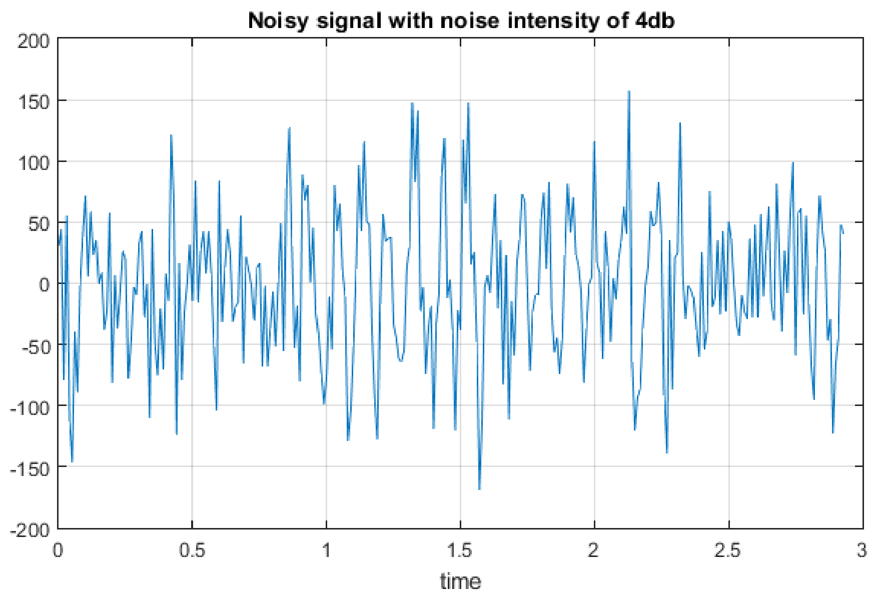

Figure 1 Noisy signal with noise intensity 4db, frequencies are $f_1 = 10Hz, f_2 = 10.5Hz$

As we see from the graph, most part of signal is distorted with noise, so that identification of components and frequencies should look very difficult, for comparison point of view, we used following three classical method 1.MUSIC(multiple signal classification) algorithm, 2.Eig(Eigenvector) algorithm 3.Periodogram(classical nonparametric) algorithm. MUSIC and Eigenvector methods are related to subspace pseudospectrum estimation and main principle is Eigenvalue decomposition of correlation matrix, and from signal and noise subspace they are trying to identify frequencies, first let us consider MUSIC method which in matlab could be written as

```
[S,f] = pmusic(x,4,[],100);
hps = dspdata.pseudospectrum(S,'Fs',100);
figure; plot(hps);
```

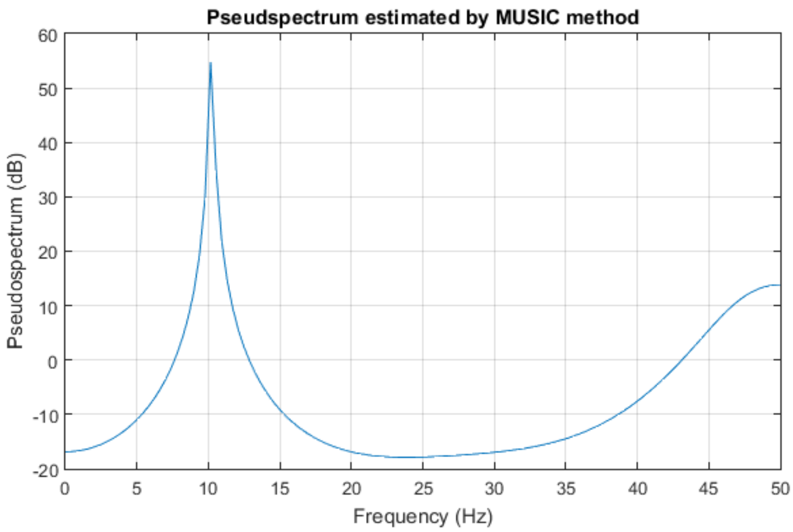

Figure 2 Pseudospectrum estimated by MUSIC method, Noisy signal with noise intensity 4db, frequencies and amplitudes are same

As we see, MUSIC method was not able to identify hidden frequency, even in case of when frequencies difference satisfy separable principle, now let us consider Eigenvector method

```
[S,f] = peig(x,4,[],100);
hps = dspdata.pseudospectrum(S,'Fs',100);
figure; plot(hps);
```

Result of Eigenvector analysis will be presented on figure (34), MUSIC and Eigenvector showed ones again that there were not able to resolve two closed space frequencies, which is obviously indicator of their low resolution ability.

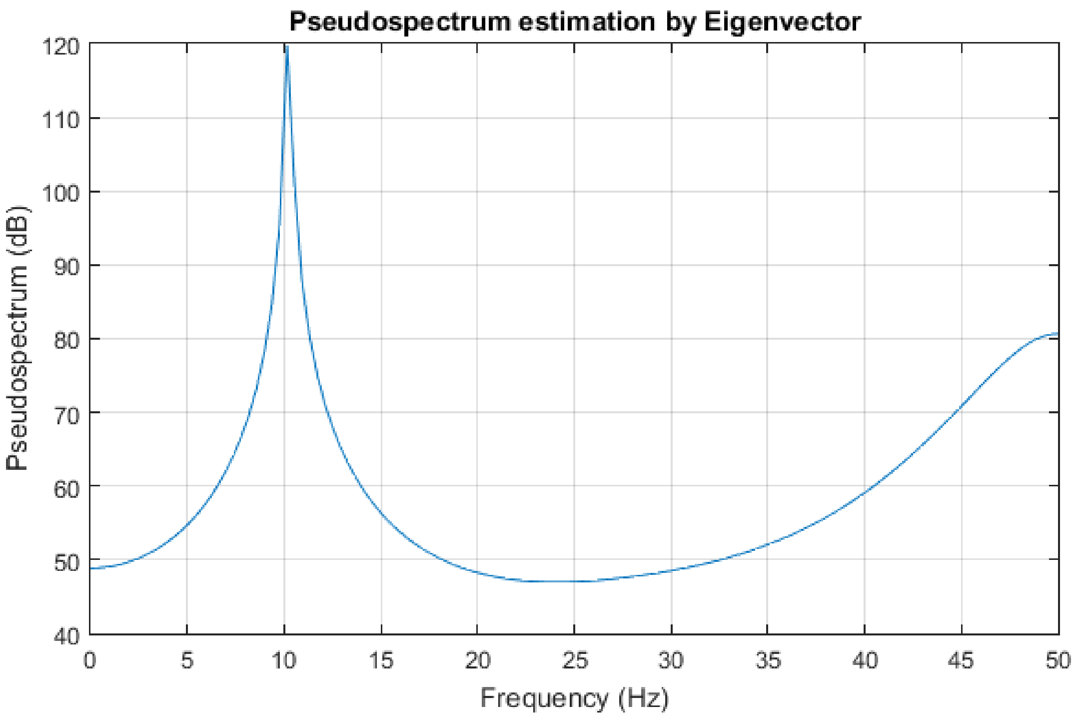

Figure 3 Pseudospectrum estimation by Eigenvector method, Noisy signal with noise intensity 4db, frequencies and amplitudes are same

Now let us consider Power spectrum estimated by periodogram method, periodogram in Matlab programming software can be written as

```
[pxx,f]=periodogram(x, [], [], fs);
plot (f,pxx)
```

here x- is time series to be analyzed, as a default we used rectangular spectral window, fs- represent as sampling frequency, pxx is a vector of estimated power spectrum and f- corresponding vector of frequencies, result of power spectrum estimation is given on figure 35

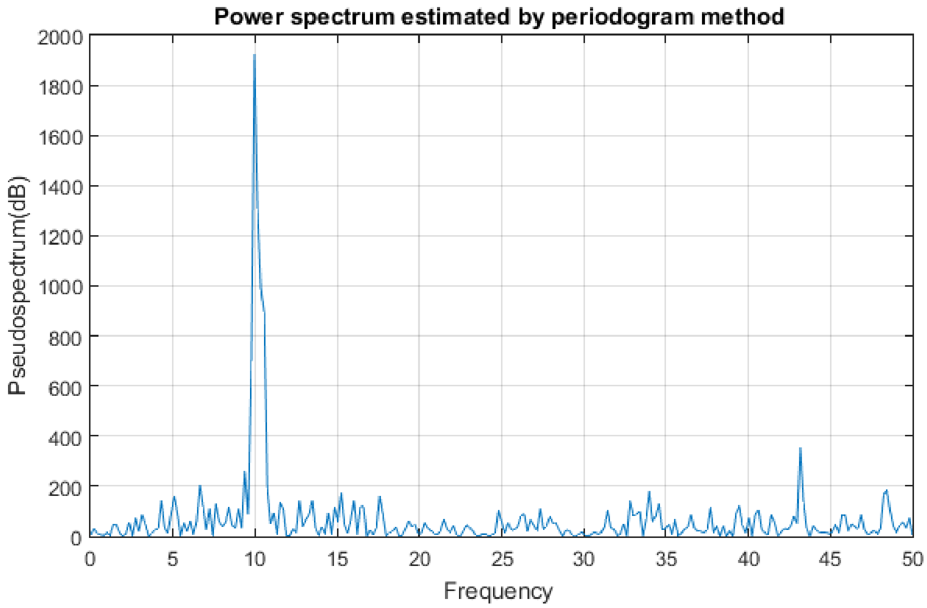

Figure 4 Power spectrum estimation by periodogram method, Noisy signal with noise intensity 4db,$f_1 = 10Hz, f_2 = 10.5Hz$

Before we start analyzing this signal by our new method, let mention several words related to Figure (35), not only periodogram was not able to discover second component, also we can easily see high intensity of noise in Power spectrum estimation, different from previous example, one reason of losing peak is that because noise intensity to signal is 4dB, that means variation of equally distributed independent random values range is equal to ± 10,therefore one of peak could be covered also by noise, now let us consider data matrix X with dimension of $185X110$, which means that after singular value decomposition , dimension of left singular matrix is equal to $185X185$ and dimension of right singular matrix $110X110$, concatenation of first and third singular vector gave us following pseudospectrum estimation result which is given to below figure,

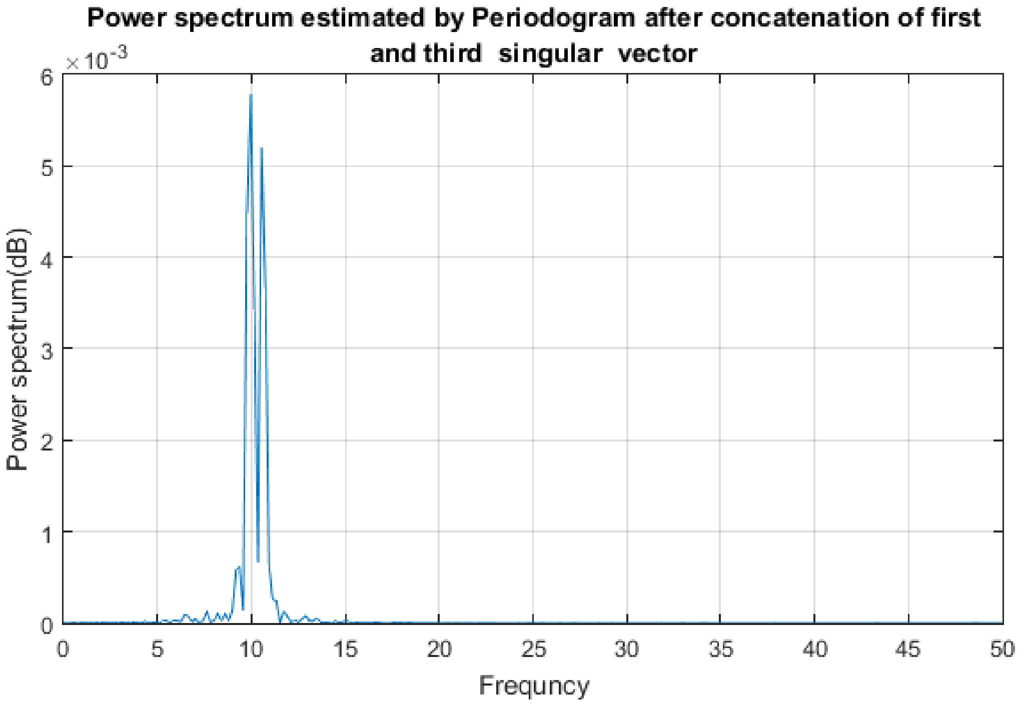

Figure 5 Power spectrum by periodogram after concatenation of first and third singular vectors

Concatenation of singular vectors while keeping same sampling frequency, increased new total observation time (time of singular sweeping) to $T_{new} = 3.7sec$, for original time series, resolution frequency was $0.34Hz$, while for new one it is equal to $0.27Hz$ which gave us possibility finding of hidden periodicities, both peak is reliable found, which can be also proved by comparison of amplitude of second larger peak to amplitude of its sidelobe. We can easily check that it satisfies separation condition (Datuashvili, Mert, & Milnikov, 2014)

$$S(f) = \frac{P(f_j)}{P(f_i)} \geq 10^{0.3} \ (i \neq j)$$

Now we can see pseudospectral structure of concatenated time series by Histogram method, which represents as a distribution of peaks of estimated Power spectrum from the new time series, Histogram obviously indicates two significantly isolated peak from the rest noise part

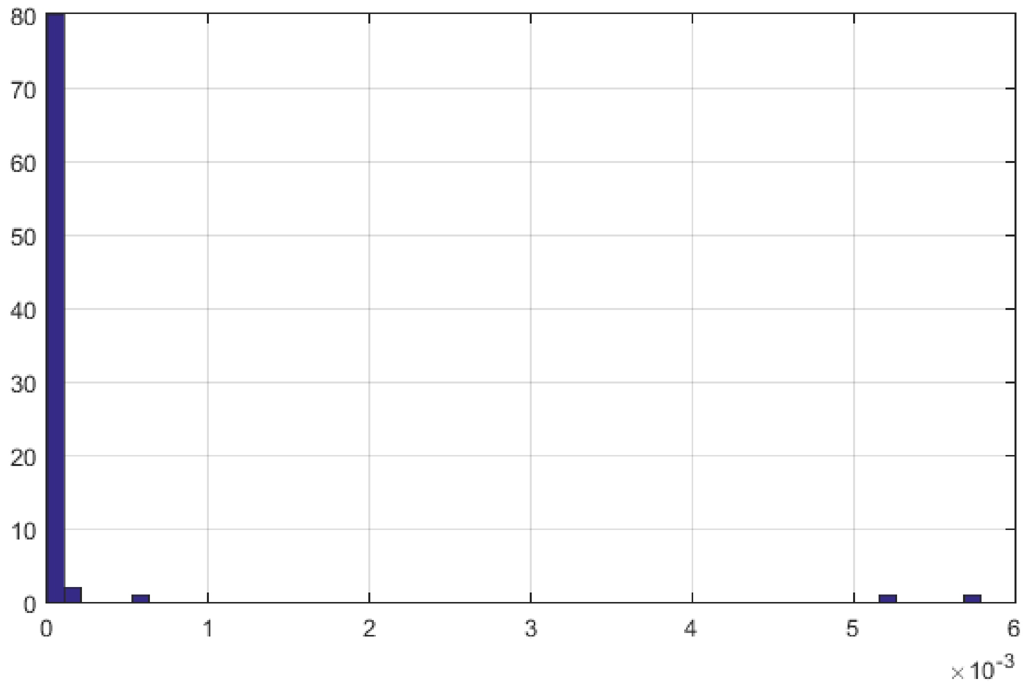

Figure 6 Histogram of distribution of peaks from the estimated Power spectrum

Finally estimation of frequencies is easy part and can be implemented by specially written matlab script, which will be introduced in Appendix B part. For given example, estimated frequencies are $f_1 = 9.9609\text{Hz}, f_2 = 10.5469Hz$, to check that estimated frequencies represent statistically accepted and reliable values, let consider regression model based on estimated and real values of frequencies. First let us consider regression for estimated values

Regression Statistics	
Multiple R	0.564528122
R Square	0.318692001
Adjusted R Square	0.309262132
Standard Error	48.99249629
Observations	294

Table 1 Regression analysis for estimated frequencies in case of high intensity noise. Difference between frequencies is greater than resolution frequency

ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	324477.55	81119.39	33.79601748	3.89675E-23
Residual	289	693676.4961	2400.265		
Total	293	1018154.046			

Table 2 Anova analysis for the estimated frequencies in case of high intensity of noise

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-1.621848083	2.857517704	-0.56757	0.570766012
α_1	38.73152313	4.087735187	9.475057	1.00555E-18
β_1	11.36050279	4.076441161	2.786868	0.005674446
α_2	25.07853108	4.079035174	6.148153	2.59889E-09
β_2	16.93307867	4.08468556	4.145504	4.45981E-05

Table 3Evaluation of regression mode for the estimated frequencies. Same model with high intensity of noise

Estimated coefficients will give us following amplitudes $A_1 = 40.36$,A_2= 30.25, which is definitely close to real values of amplitudes, now consider regression analysis for the real frequencies

Regression Statistics	
Multiple R	0.596000472
R Square	0.355216563
Adjusted R Square	0.346292225
Standard Error	47.66117675
Observations	294

Table 4 Regression analysis for the real frequencies

ANOVA					
	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	361665.1807	90416.3	39.80312642	1.51024E-26
Residual	289	656488.8654	2271.588		
Total	293	1018154.046			

Table 5 Anova analysis in case of high intensity of noise. Real frequencies case

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	-1.602817021	2.779971193	-0.57656	0.564686276
α_1	43.2471426	4.022730687	10.75069	6.72247E-23
β_1	-3.370644475	4.029194565	-0.83656	0.403533698
α_2	34.66073449	4.026170587	8.608859	4.83444E-16
β_2	4.561553768	4.025352492	1.133206	0.258066952

Table 6 evaluation of regression model for time series with high intensity of noise. Difference between frequencies is greater than resolution frequency

And for real frequencies, calculated amplitudes are $f_1 = 43.37, A_2 = 34.95$,as we see even for real values of frequencies, amplitudes are still difference from real ones and also regression did show small value of Multiple R, which is of course caused by high level of noise, our proposed method showed significant power over existing methods in two aspects 1. Power spectrum estimated from the concatenation of first and third singular vector showed definitely separated two significant peaks, which was not occurred during analysis of presented methods in this thesis 2. Original time series power spectrum picture contained a lot of noise, it was due to existence of high intensity random variables, after singular value decomposition of Hankel matrix and vertical concatenation of first and third singular vectors, level of noise was noticeably decreased ,that interesting fact can be explained by methods itself ,which plays as a filter role.

Conclusion

The main scientific and practical results obtained in the dissertation:

1 Has been developed new method for estimation Pseudospectrum of time series consist of periodic deterministic components, that significantly increase resolving capacity
2 Has been Proposed new approach of time series approximation, based on low rank tensor approximation
3 Has been Proposed new iterated method for singular value decomposition of data matrix
4 Has been introduced terms of order of low rank approximation and time of singular sweeping
5 Has been shown that left and right singular vectors of data matrix and original time series have equal pseudospectral structure
6 Has been shown that by concatenation of singular vectors, we can increase time of singular sweeping, which at the same time increase statistical stability and resolution of pseudospectral estimation
7 Based on examples considered in the thesis, Has been shown practical and statistical reliability of given result
8 Based on practical examples made by special programming software, comparative analysis of newly proposed method and existed methods has been done , which has been provided effectiveness of new method

Published articles

1. Datuashvili D., Mert C., Milnikov A. (2014). New Approach to Detecting Deterministic Periodic Components in Noise. Proceedings of the 5th international Conferences on Circuits, Systems, Communications, Computers and Applications (CSCCA'15), Salerno, Italy, p. 70-75
2. Milnikov A., Datuashvili D. (2014). Iterated SVD for Improving Spectral Resolution of Nonstationary Signal. *IBSU,Journal of Technical Science and Technologies*, Vol 3, No 1 p 5-8
3. Datuashvili D. (2014). Improving Spectral Resolution of a Stationary Signal Using Singular Value Decomposition, IBSU,Journal of Technical Science and Technologies, Vol 3, No 2, p.31-34